

$$A(-2, k), B(4, m) \rightarrow m_{AB} = -\frac{1}{4} \Rightarrow m_{AB} = \frac{m-k}{4-(-2)} = -\frac{1}{4} \Rightarrow m-k = -2$$



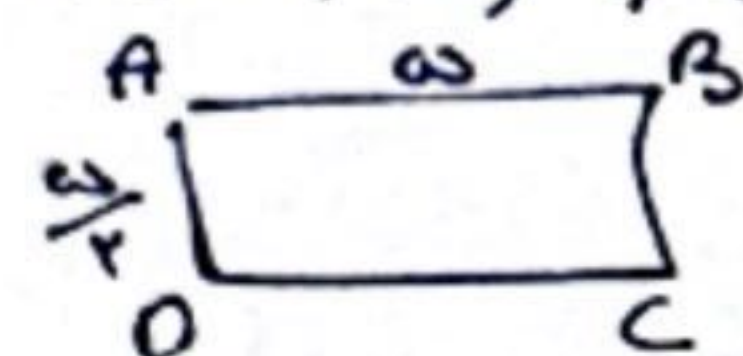
$$\Rightarrow AB = \sqrt{(4+2)^2 + (m-k)^2} \Rightarrow AB = \sqrt{6^2 + (-2)^2} = \sqrt{40} = 2\sqrt{10}$$

$$S = (\sqrt{40})^2 = 40$$

پاسخ

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$$A(-1, k), B(3, 1), C(u, y), D(-1-u, y+2)$$



$$AB = \sqrt{(4)^2 + (1-k)^2} \Rightarrow AB = \sqrt{16 + (1-k)^2} = \omega$$

$$AB = CD = \omega \Rightarrow \sqrt{(y+2-1)^2 + (-1-u-3)^2} = \sqrt{9 + (-2u-4)^2} = \omega$$

$$\Rightarrow 9 + 4u^2 + 16 + 16u = \omega^2 \Rightarrow 4u^2 + 16u + 25 = \omega^2 \Rightarrow \Delta = 16\omega^2 \Rightarrow u = \frac{-16 \pm \sqrt{16\omega^2}}{8} = \frac{-16 \pm 4\omega}{8}$$

$$\Rightarrow u = \frac{-4 \pm \omega}{2} \rightarrow u_1 = \frac{-4 + \omega}{2}, u_2 = \frac{-4 - \omega}{2}$$

$$\Rightarrow y = -3, y = -1$$

$$C(\frac{\omega}{2} - 1, -1), D(-\frac{\omega}{2} - 1, 1) \Rightarrow BC = AD = \sqrt{1 + \frac{\omega^2}{4}} = \sqrt{\frac{4 + \omega^2}{4}} = \frac{\sqrt{4 + \omega^2}}{2}$$

$$2 \times \omega + 2 \times \frac{\sqrt{4 + \omega^2}}{2} = 10 + \omega \Rightarrow \omega = 4$$

پاسخ

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$$m = \tan \alpha \Rightarrow m = \tan 45^\circ \Rightarrow m = 1$$

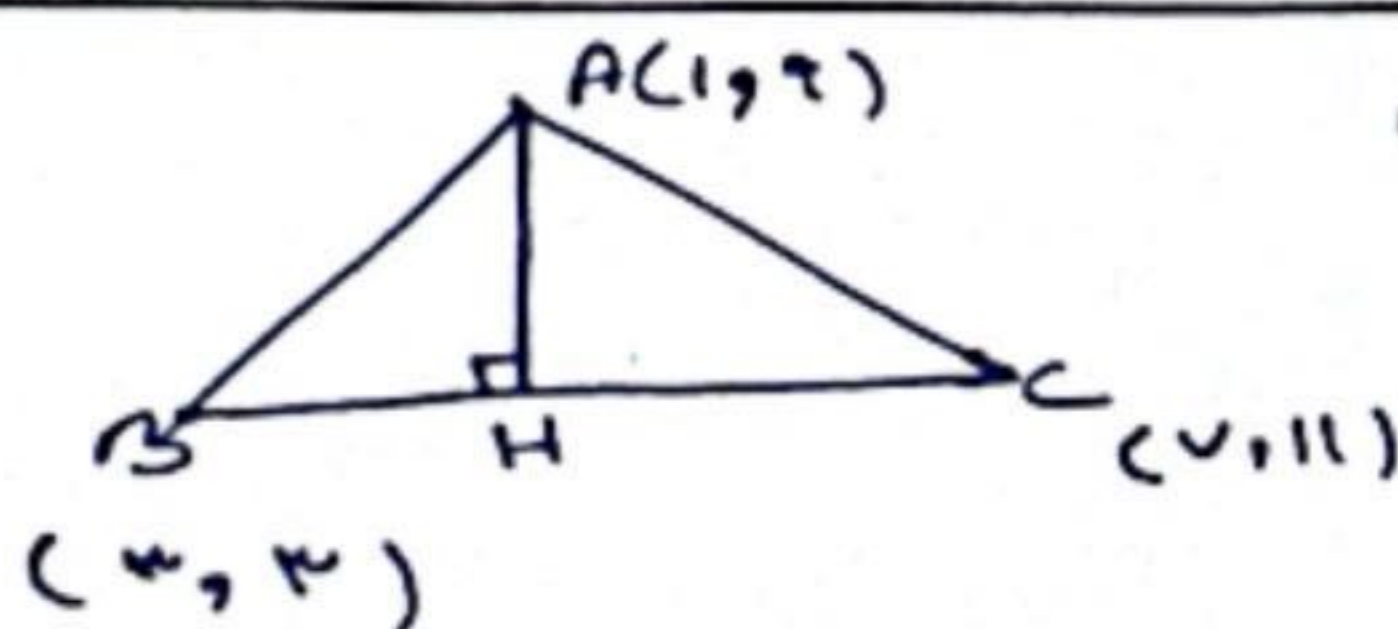
$$2mu + (m^2 - 1)y = 2 \rightarrow m = \frac{2m}{m^2 - 1} = 1 \Rightarrow -2m = \sqrt{3}m^2 - \sqrt{3} \Rightarrow \sqrt{3}m^2 - \sqrt{3} + 2m = 0$$

$$\Rightarrow m^2 - 1 + 2m = 0 \Rightarrow (m+1)(m-1) = 0 \Rightarrow m_1 = -1, m_2 = 1$$

$$\alpha - \beta = \frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{1 + 4\sqrt{3}}}{\sqrt{3}} = \frac{1 + \sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3} + 3}{3}$$

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$$m_{BC} = \frac{1-1}{5-3} = 0$$

$$y - y_0 = m(x - x_0)$$

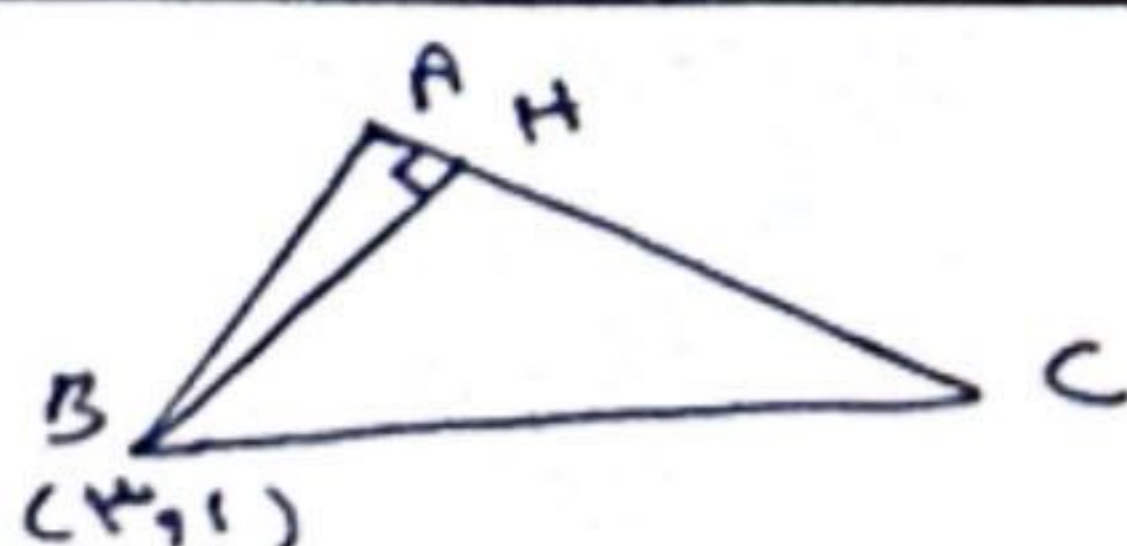
$$y - 1 = 0(x - 3) \Rightarrow y = 1 \Rightarrow y - 2x + 3 = 0$$

$$d = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}} \Rightarrow d = \frac{|1 - 2 + 3|}{\sqrt{1 + 4}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

$$\Rightarrow d = \frac{2\sqrt{5}}{5}$$

پاسخ

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$$AB: (y - 2) = -1(x - 1) \Rightarrow y - 2 = -x + 1 \Rightarrow x + y - 3 = 0$$

$$BC: 2y - 5x = -19$$

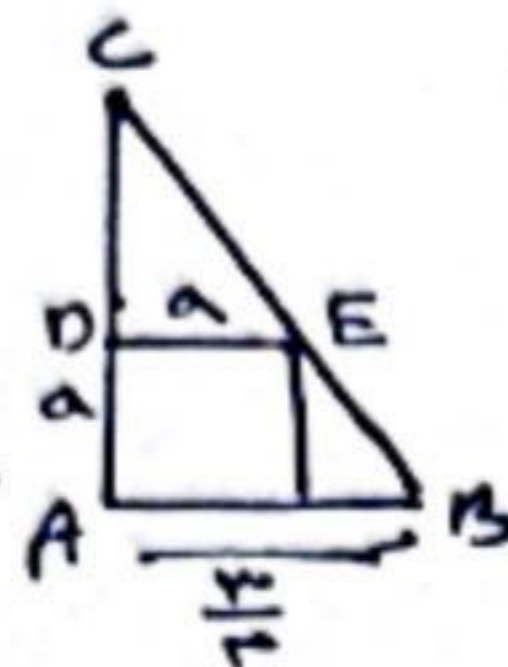
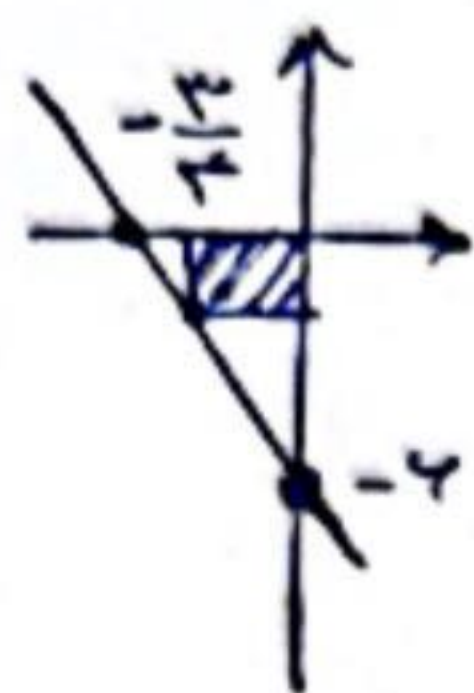
$$-11u = -33$$

$$u = 3, y = 1 \rightarrow B(3, 1)$$

$$AC: 4y - 3x = 14 \Rightarrow AC: 4y - 3x - 14 = 0 \Rightarrow d = \frac{|4 - 9 - 14|}{\sqrt{16 + 9}} = \frac{19}{5}$$

پاسخ

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مستطیل ADEF $\rightarrow$ $AD \parallel DE$ $\rightarrow$ $\frac{AD}{AC} = \frac{DE}{AB}$

$$\Rightarrow \frac{y-a}{y} = \frac{\frac{y}{2}}{\frac{y}{2}} = \frac{y-a}{y} \Rightarrow 1 - \frac{a}{y} = \frac{y-a}{y} \Rightarrow y-a = y-a \Rightarrow y-a = 1$$

$$\Rightarrow a = \frac{y}{2}$$

مساحت = $a\sqrt{2} = \frac{y\sqrt{2}}{2}$

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$\begin{cases} y-a=1 \rightarrow y=a+1 \\ y-au=1 \rightarrow y=au+1 \end{cases} \xrightarrow{m=m'} \frac{1}{a} = a \Rightarrow a^2=1 \Rightarrow a=\pm 1$

$\rightarrow a=1 \Rightarrow y=u \Rightarrow y=u+1 \rightarrow (1, 2) \checkmark$

$\rightarrow a=-1 \Rightarrow y=-u+2, y=-u+1 \rightarrow (1, 2) \checkmark$

$y = \frac{|c-c'|}{\sqrt{a^2+b^2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \Rightarrow \omega = \sqrt{u^2+y^2} \Rightarrow r\omega = u^2 + \left(\frac{\sqrt{2}}{2}\right)^2 \Rightarrow u^2 = \frac{r^2-1}{2} \Rightarrow u = \frac{\sqrt{r^2-1}}{2}$

$\Rightarrow S = uy = \frac{\sqrt{2}}{2} \times \frac{\sqrt{r^2-1}}{2} = \frac{\sqrt{2(r^2-1)}}{4} \rightarrow$ جواب

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$u-3y=1 \Rightarrow 3y=u-1 \Rightarrow y = \frac{1}{3}u - \frac{1}{3}$ $A(1, 0), B(\frac{1}{3}, \frac{1}{3})$

$d = \frac{|1-1|}{\sqrt{1+9}} = \frac{3\sqrt{10}}{10}$

$m_{AC} = -3 \Rightarrow y = -3(u-1) = -3u+3$

نقطه C $\rightarrow \frac{1}{3}u - \frac{1}{3} = -3u+3 \Rightarrow \frac{10}{3}u = 10 \Rightarrow u=3, y=0$

$\begin{bmatrix} 3 & 1 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \rightarrow 0, 9 - 3, 4 = -2, 1$

$S = \frac{1}{2} \times 3 \times 1 = \frac{3}{2} \rightarrow$ جواب

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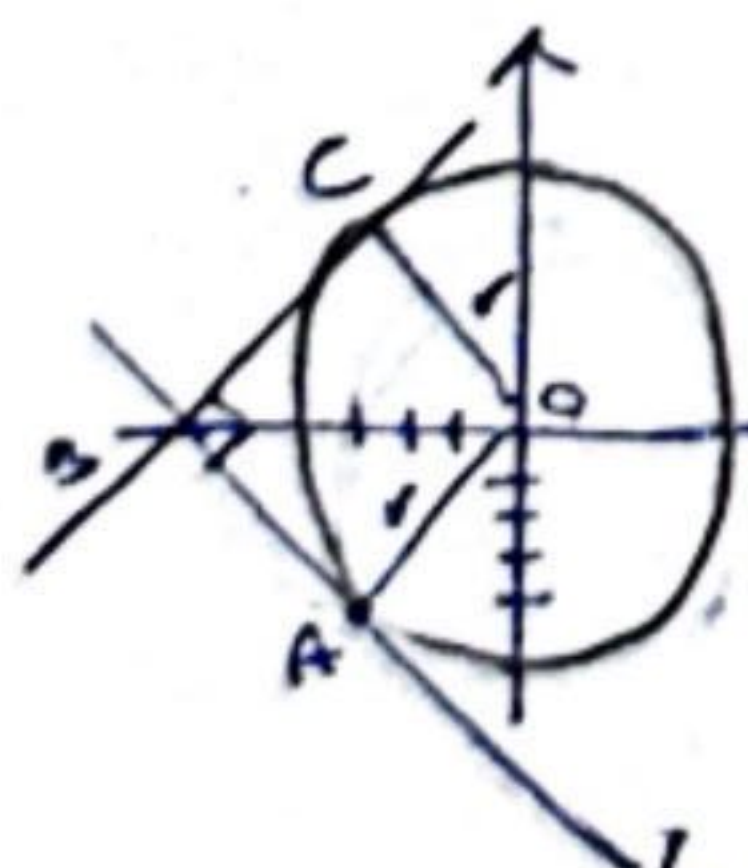


$m_{\Delta} = \sqrt{3} \Rightarrow m_{\Delta} = \frac{a-b}{-\frac{1}{4}+\frac{1}{4}} \Rightarrow m_{\Delta} = \frac{a-b}{-\frac{1}{4}} = \sqrt{3} \Rightarrow a-b = -\frac{\sqrt{3}}{4}$

$\sqrt{\left(-\frac{\sqrt{3}}{4}\right)^2 + \left(-\frac{1}{4}\right)^2} = \sqrt{\frac{3}{16} + \frac{1}{16}} = \sqrt{\frac{4}{16}} = \frac{1}{2} = \frac{1}{2} \rightarrow$ فاصله = a

$a\sqrt{2} = \text{مساحت} \Rightarrow \frac{1}{2} \times \sqrt{2} = \frac{\sqrt{2}}{2} \rightarrow$ جواب

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$\Rightarrow \sqrt{(a+3)^2 + (a+3)^2} = \sqrt{2a^2} = \omega = r$

$m_{OA} = \frac{3}{4} \Rightarrow m_{BC} = \frac{3}{4} \Rightarrow BC \Rightarrow y = \frac{3}{4}u + b \Rightarrow 3y - 4u - 4b = 0$

فاصله از O تا BC برابر است $\Rightarrow d = \frac{|3y - 4u - 4b|}{5} = \omega \Rightarrow -4b = 2a \Rightarrow b = \frac{2a}{4}, b = \frac{a}{2}$

$BC \Rightarrow y = \frac{3}{4}u + \frac{a}{2} \Rightarrow BC = -4u + 3y - 2a = 0$

$m_{AB} = -\frac{3}{4} \rightarrow y + \frac{3}{4} = -\frac{3}{4}(u + \frac{3}{4}) \Rightarrow y = -\frac{3}{4}u - \frac{3}{4}$

$\Rightarrow AB \Rightarrow 3u + 4y + 3 = 0 \Rightarrow u = -\frac{4a-10}{-12-9} = -\frac{4a-10}{-21-9} = \frac{10-4a}{-30}$

$\Rightarrow uy = -1 \times -\frac{1}{2} = \frac{1}{2}$

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