

۱. معادله گویه معرفت خط گذرنده از دو نقطه $A(4, -2)$ و $B(2, 8)$

شیب $AB \rightarrow \frac{8 - (-2)}{2 - 4} = \frac{10}{-2} = -5$

معادله خط $y = -5x + \frac{14}{2} \rightarrow 2y - 10x + 7 = 0$

۲. مساحت ΔABC را با $A(3, 1)$, $B(-5, 7)$, $C(1, 3)$ محاسبه کنید.

شیب $AB = \frac{7-1}{-5-3} = -\frac{6}{8} = -\frac{3}{4}$

معادله خط $AB: y = -\frac{3}{4}x + \frac{19}{4}$

مساحت $\Delta ABC = \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

$\Delta ABC = \frac{1}{2} |3(7-3) + (-5)(3-1) + 1(1-7)| = \frac{1}{2} |12 - 10 - 6| = \frac{1}{2} | -4 | = 2$

۳. یک دایره با مرکز $O(0, 0)$ و شعاع r به خط $y = \frac{1}{2}x + \frac{1}{2}$ مماس است. شعاع r را بیابید.

فاصله از مرکز دایره تا خط:

$r = \frac{|-\frac{1}{2} \cdot 0 + 1 - \frac{1}{2}|}{\sqrt{\frac{1}{4} + 1}} = \frac{|\frac{1}{2}|}{\sqrt{\frac{5}{4}}} = \frac{1/2}{\sqrt{5}/2} = \frac{1}{\sqrt{5}}$

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~~(M+1)x + 1~~

$$(M-5)y = (M+1)x + 1$$

$$y = \frac{(M+1)x + 1}{(M-5)} \rightarrow \frac{M+1}{M-5}$$

$$-(5-M)y = (M-5)x + 1$$

$$y = \frac{(M-5)x + 1}{-(5-M)} \rightarrow \frac{M-5}{M-5}$$

$$\frac{M+1}{M-5} = -\frac{M-5}{M-5}$$

$$(M+1)(M-5) = -(M-5)(M-5)$$

$$M^2 - 5M + M - 5 = -M^2 + 10M - 25$$

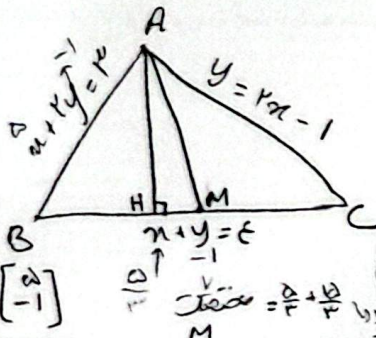
$$M^2 - 4M - 5 = -M^2 + 10M - 25$$

$$2M^2 - 14M + 20 = 0$$

$$M^2 - 7M + 10 = 0$$

$$(M-2)(M-5) = 0$$

$$149 - 5(5)(11) < 0$$



$$-x + y - \varepsilon = x + y - r$$

$$y = -1 \rightarrow x = 0$$

$$x + y - \varepsilon = y - r + 1$$

$$r + 1 = \varepsilon \rightarrow r = \varepsilon - 1$$

$$y - r + 1 = x + y - r$$

$$y + r - \varepsilon = 0$$

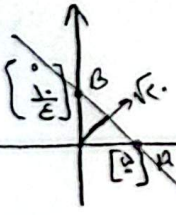
$$y + 1 - \varepsilon = y + 1 - \varepsilon$$

$$\frac{AH}{BC} = \frac{1}{2}$$

$$AM = \sqrt{A^2 - \left(\frac{1}{2}\right)^2} = \sqrt{1 - \frac{1}{4}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$$

$$y = -\frac{1}{r}x + B$$

$$r = \frac{1}{\varepsilon} \rightarrow B = -\frac{1}{\varepsilon}$$



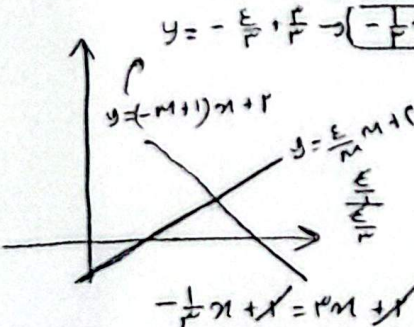
$$y = -\frac{1}{r}x + \frac{1}{\varepsilon}$$

$$\frac{|0 + 0 - B|}{\sqrt{1 + \frac{1}{\varepsilon^2}}} = \frac{B}{\sqrt{1 + \frac{1}{\varepsilon^2}}} = \frac{B}{\frac{\sqrt{\varepsilon^2 + 1}}{\varepsilon}} = \frac{B \varepsilon}{\sqrt{\varepsilon^2 + 1}}$$

$$B = \frac{1}{\varepsilon}$$

$$A \Rightarrow y = -\frac{1}{r}(0) + \frac{1}{\varepsilon} = \frac{1}{\varepsilon}$$

$$B, A \text{ are } \rightarrow \sqrt{(0 - 0)^2 + \left(\frac{1}{\varepsilon} - 0\right)^2} = \frac{1}{\varepsilon}$$



$$y = -\frac{1}{r}x + \frac{1}{\varepsilon} \rightarrow y = -\frac{1}{r}x + \frac{1}{\varepsilon}$$

$$my - \varepsilon n = cm$$

$$my = \varepsilon m + cm$$

$$y = \frac{\varepsilon}{m}x + c$$

$$y = -\frac{m}{m+1}x + \frac{r}{m+1}$$

$$y = x(-m+1) + r$$

$$-m+1 = -\frac{m}{\varepsilon}$$

$$m-1 = \frac{m}{\varepsilon}$$

$$\varepsilon m - \varepsilon = m$$

$$r m - \varepsilon = 0$$

$$r m = \varepsilon \rightarrow m = \frac{\varepsilon}{r}$$

$$m = (r-1) \rightarrow y = rx - r$$

$$r m = \frac{r A + r A'}{r} = r = \frac{0 + r A'}{r}$$

$$A \rightarrow \begin{bmatrix} 0 \\ -r \end{bmatrix}$$

$$r A' = \varepsilon$$

$$y m = \frac{y A + y A'}{r} = -r = \frac{-r + y A'}{r}$$

$$A' \begin{bmatrix} \varepsilon \\ -1 \end{bmatrix}$$

$$-r = \frac{-r + y A'}{r}$$

$$y = \frac{1}{r}x - r$$

$$r b - a, \text{ where } A'(a, b) = r, y = x + a, \text{ where } A = (1, r)$$



$$y = x + a$$

$$y = -x + r$$

$$-x + r = x + a$$

$$-r = a \rightarrow a = -r$$

$$\frac{1 + r A'}{r} = -1$$

$$1 + r A' = -r$$

$$r A' = -r - 1 = a$$

$$y A' = b$$

$$r b - a$$

$$r(y) - (-r)$$

$$1r + r = 1 \rightarrow 1$$