

سوال (۱)

$A(4, -4)$ و $B(2, 8)$

وسط پاره خط: $(\frac{4+2}{2}, \frac{-4+8}{2}) = (3, 1)$

$\begin{cases} 2a+b=8 \\ 4a+b=-4 \end{cases} \rightarrow b=12, a=-7 \Rightarrow \text{شیب} = -7 \Rightarrow \text{شیب عمودش} = \frac{1}{7}$

$\rightarrow$ معادله محور: $y-1 = \frac{1}{7}(x-3) \rightarrow y-1 = \frac{1}{7}x - \frac{3}{7} \rightarrow 7y-7 = x-3 \rightarrow 7y = x+4$

$A(3, 1), B(-4, 5), C(-6, 3)$

سوال (۲) AH ارتفاع وارد بر ضلع BC

BC $\rightarrow C-B: (-1, -4), 3-5 = (-4, -2) \rightarrow \sqrt{1+16} = \sqrt{17} = \underline{5}$

AH $\rightarrow H: (\frac{5}{5}, \frac{-1}{5}) \rightarrow \frac{\sqrt{4+1}}{5} = \frac{\sqrt{5}}{5} = \frac{1}{5} = \underline{2}$

AH ارتفاع از قعر BC $\rightarrow 5-2 = \underline{3}$

$7y-x=7, 4y=2x+7$ $S_{\text{دایره}} = ?$

$4y=2x+7 \rightarrow 4y-2x=7 \Rightarrow a=2$

$7y-x=7 \rightarrow 7y-2x=14$

$2r = \frac{|14-7|}{\sqrt{1+4}} = \frac{|7|}{\sqrt{5}} \rightarrow 2r = \frac{7}{\sqrt{5}} \rightarrow r = \frac{7}{2\sqrt{5}}$

$S = \pi r^2 = \pi \times (\frac{7}{2\sqrt{5}})^2 \rightarrow$

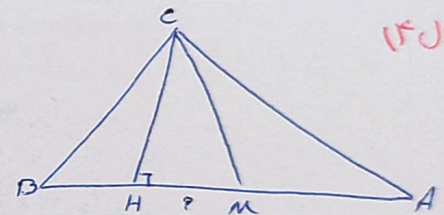
$S = \frac{49}{5} \pi$

سوال (۳)

$A(2, 3), B(4, 1), C(-1, -3)$

میان M: $(\frac{2+4}{2}, \frac{3+1}{2}) = (3, 2)$

$y_{AB} = -x+5$
 $\downarrow$
 $y_{CH} = x-2$
 $\begin{cases} 5-x=y \\ x-2=y \end{cases} \Rightarrow 5-x=x-2 \rightarrow 7=2x \rightarrow x=3.5$
 $\Rightarrow H: (3.5, 1.5)$



سوال (۴)

$\Rightarrow MH = \sqrt{(3.5-3)^2 + (1.5-2)^2} = \sqrt{(0.5)^2 + (-0.5)^2} = \sqrt{0.25+0.25} = \sqrt{0.5} = \frac{\sqrt{2}}{2}$

$(3m-2)x + (2-m)y - 7 = 0 \perp (m+1)x - (2m-4)y + 1 = 0$

$(3m-2)x + (2-m)y - 7 = 0 \rightarrow$ شیب: $a = -\frac{3m-2}{2-m}$

$(m+1)x - (2m-4)y + 1 = 0 \rightarrow$ شیب: $a' = \frac{m+1}{2m-4}$

موازی بودن: $aa' = -1 \rightarrow (-\frac{3m-2}{2-m})(\frac{m+1}{2m-4}) = 1 \rightarrow \frac{(3m-2)(m+1)}{(m-2)(2m-4)} = 1 \rightarrow (3m-2)(m+1) = (m-2)(2m-4)$

$\rightarrow 3m^2 + m - 2 = 2m^2 - 4m + 8 \rightarrow m^2 + 5m - 10 = 0 \rightarrow \Delta = 25 + 40 = 65 \rightarrow m = \frac{-5 \pm \sqrt{65}}{2}$

سوال (۵) m = ?

معادله ریشه حقیقی ندارد پس مقدار حقیقی برای m وجود ندارد در خط

$\begin{cases} AB: x+y=3 \\ AC: y=2x-1 \\ BC: x+y=4 \end{cases}$ $\frac{AM}{AH} = ?$ $BC \rightarrow \frac{2}{3}a+b=\frac{4}{3} \rightarrow 2a+3b=4$
 $\frac{2}{3}a+b=-1 \rightarrow 2a+3b=-3$
 $\rightarrow 2b=7 \rightarrow b=\frac{7}{2} \rightarrow a=-1 \rightarrow x+y=3 \rightarrow x+2=3 \rightarrow x=1 \rightarrow y=2 \rightarrow H: (1, 2) \rightarrow AH = \sqrt{2}$

سوال (۶)

$x+y=3$
 $y=2x-1$
 $\rightarrow x+2(2x-1)=3 \rightarrow x+4x-2=3 \rightarrow 5x=5 \rightarrow x=1 \Rightarrow A: (1, 1)$
 $y=2(1)-1=1$

$\frac{AM}{AH} = \frac{\frac{5\sqrt{2}}{3}}{\sqrt{2}} = \frac{5}{3}$

$x+y=3$
 $x+y=4$
 $\rightarrow (x+y)-(x+y)=3-4 \rightarrow y=-1, x-1=4 \rightarrow x=5 \Rightarrow B: (5, -1)$

$y=2x-1$
 $x+y=4$
 $\rightarrow x+2(2x-1)=4 \rightarrow 3x=6 \rightarrow x=2, y=\frac{3}{2} \Rightarrow C: (\frac{5}{2}, \frac{3}{2})$

$M: (\frac{5+\frac{5}{2}}{2}, \frac{-1+\frac{3}{2}}{2}) \rightarrow M: (\frac{15}{4}, \frac{1}{4}) \Rightarrow \frac{AM}{AH} = \sqrt{(\frac{15}{4}-1)^2 + (\frac{1}{4}-1)^2} = \sqrt{\frac{169}{16} + \frac{9}{16}} = \frac{5\sqrt{2}}{2}$

سوال ۷

شیب خط: $m = -\frac{1}{2}$ نامتوا خط: $\sqrt{2}$

$m = -\frac{1}{2} \rightarrow y = -\frac{1}{2}x + b \rightarrow -\frac{1}{2}x + b - y = 0 \rightarrow -x - 2y + 2b = 0$

نامتوا خط: $\sqrt{2} \rightarrow \frac{|2b|}{\sqrt{2}} = \sqrt{2} \rightarrow 2b = 1 \rightarrow b = \frac{1}{2}$

$\Rightarrow x + 2y - 1 = 0$

AB نامتوا = $\sqrt{(1-0)^2 + (0-\frac{1}{2})^2} = \sqrt{1 + \frac{1}{4}} = \sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{2}$

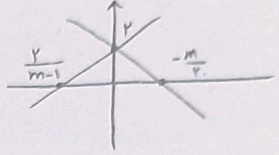
$my - 2x = 2m$, $y + mx = x + 2$ $S_A = \frac{2}{1}$

$y = \frac{2}{m-1}x + 2$

$y = (1-m)x + 2$

هر دو خط در نقطه ای
مماس و موازی

حالت مماس در دو خط
مماسی و موازی



$S_{\text{مساحت}} = \frac{1}{2} \times \frac{1}{2} \times (1 - \frac{m}{2} - \frac{2}{m-1})$

$\Rightarrow \left| \frac{m}{2} + \frac{2}{m-1} \right| = \frac{1}{2}$

$\frac{m^2 - m + 4}{2m - 2} = \frac{1}{2} \Rightarrow m^2 - m + 4 = m - 2$ (1)

$\frac{m^2 - m + 4}{2m - 2} = \frac{-1}{2} \Rightarrow m^2 - m + 4 = -m + 2$ (2)

(1) $\rightarrow m^2 - 4m + 6 = 0 \rightarrow (m-2)^2 = 0 \rightarrow m = 2$

(2) $\rightarrow m^2 + 3m - 2 = 0 \rightarrow \Delta > 0$

$\Rightarrow -3$

$y = 2x - 3$ $x(2, -2)$

سوال ۹

$x=1 \rightarrow y = 2(1) - 3 \rightarrow y = -1 \rightarrow (1, -1) \xrightarrow{\text{قرینه نسبت به } y=2} (3, -3)$

$x=0 \rightarrow y = 2(0) - 3 \rightarrow y = -3 \rightarrow (0, -3) \xrightarrow{\text{قرینه نسبت به } y=2} (4, -1)$

$(3, -3), (4, -1) \xrightarrow{\text{معادله خط}} \begin{cases} 3a+b=-1 \\ 4a+b=-3 \end{cases} \Rightarrow \begin{matrix} a=2 \\ b=-7 \end{matrix}$

$\Rightarrow$ معادله قرینه خط: $y = 2x - 9$

$A(1, 2)$ $y = x + 5$ $A'(a, b)$ $2b - a = ?$

سوال ۱۰

معادله خط عمود بر $y = x + 5$: $y = -x + 5$ شیب = -1

$y - 2 = -1(x - 1) \rightarrow y = -x + 3$

$y = x + 5$ $y = -x + 3$ $x + 5 = -x + 3 \rightarrow 2x = -2 \rightarrow x = -1$ و $y = -(-1) + 3 \rightarrow y = 4 \rightarrow (-1, 4)$
نقطه وسط AA'

$\frac{1+a}{2} = -1 \rightarrow 1+a = -2 \rightarrow a = -3$ $\Rightarrow A' : (-3, 9)$

$\frac{2+b}{2} = 4 \rightarrow 2+b = 8 \rightarrow b = 6$

$2b - a = 2(6) - (-3) = 12 + 3 = 15$