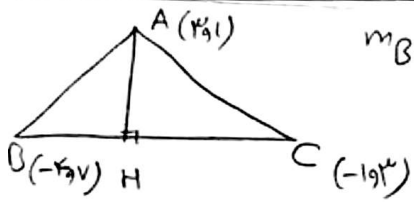


$$m = \frac{-4-1}{k-2} = \frac{-1k}{2} = -\frac{k}{2} \Rightarrow y = -\frac{k}{2}x + b \rightarrow b = \frac{k}{2} \Rightarrow y = -\frac{k}{2}x + \frac{k}{2}$$

$$M = \left(\frac{k+2}{2}, \frac{1-4}{2}\right) = \left(\frac{k}{2}+1, -\frac{3}{2}\right)$$

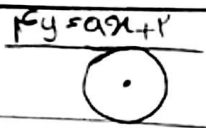
$$y = x + k$$



$$m_{BC} = \frac{3-2}{-1-k} = \frac{-1}{k+1} \Rightarrow y = -\frac{1}{k+1}x + b \rightarrow b = \frac{1}{k+1} \Rightarrow y = -\frac{1}{k+1}x + \frac{1}{k+1}$$

$$AH = \frac{|(-k \times 1) + (-k \times 2) + 1|}{\sqrt{(-k)^2 + (-1)^2}} = \frac{|-3k-1|}{\sqrt{k^2+1}} = \frac{10}{\sqrt{5}} = 2\sqrt{5}$$

$$BC = \sqrt{(-k)^2 + (2-3)^2} = \sqrt{k^2+1} \Rightarrow BC - AH = \sqrt{k^2+1} - 2\sqrt{5}$$

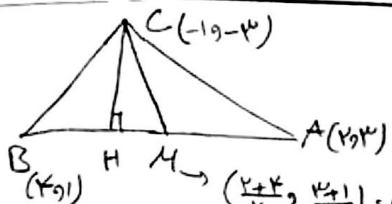


$$\Rightarrow \frac{a}{k} = \frac{1}{2} \rightarrow a = \frac{k}{2}$$

$$\begin{cases} 2x - ky + 2 = 0 \\ -x + 2y - 2 = 0 \end{cases} \xrightarrow{x=2} \begin{cases} 4 - 2k + 2 = 0 \\ -2 + 4 - 2 = 0 \end{cases} \Rightarrow 2k = 6 \Rightarrow k = 3$$

$$S_{\text{circle}} = \left(\frac{10}{\sqrt{5}}\right)^2 \pi = \frac{40}{\pi}$$

$$\text{مساحت دایره} = \frac{|2-1k|}{\sqrt{k^2+4}} = \frac{12}{\sqrt{5}} = \frac{4}{\sqrt{5}} \Rightarrow \frac{4}{\sqrt{5}} = \frac{r}{\sqrt{5}} \Rightarrow r = 4$$



$$m_{AB} = \frac{3-1}{2-1} = 2 \Rightarrow m_{CH} = -\frac{1}{2} \Rightarrow y = -\frac{1}{2}x + b \rightarrow b = \frac{3}{2} \Rightarrow y = -\frac{1}{2}x + \frac{3}{2}$$

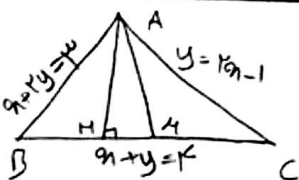
$$m_{BC} = \frac{-3-1}{-1-1} = 1 \Rightarrow y = x + b \rightarrow b = -2 \Rightarrow y = x - 2$$

$$-x + 3 = x - 2 \Rightarrow x = \frac{5}{2}, y = \frac{1}{2}$$

$$MH = \sqrt{\left(\frac{5}{2} - 1\right)^2 + \left(\frac{1}{2} - 2\right)^2} = \sqrt{\left(\frac{3}{2}\right)^2 + \left(-\frac{3}{2}\right)^2} = \frac{3\sqrt{2}}{2}$$

$$m_1 = -\frac{2m-2}{a-m} \Rightarrow \frac{a-m}{2m-2} \text{ و } m_2 = -\frac{m+1}{-(2m-2)} = \frac{m+1}{2m-2} \Rightarrow \frac{m+1}{2m-2} = \frac{a-m}{2m+2}$$

$$2m^2 + am + 2 = -2m^2 + 4m - 2 \Rightarrow 4m^2 - 3m + 4 = 0 \Rightarrow \Delta < 0 \Rightarrow \text{فاصله بین مماس ها}$$



$$\begin{cases} x-1 = 1-x \rightarrow x_C = \frac{1}{2}, y_C = \frac{1}{2} \\ x-1 = \frac{1-x}{2} \rightarrow x_A = 1, y_A = 1 \\ \frac{1-x}{2} = 1-x \rightarrow x_B = 0, y_B = 0 \end{cases}$$

$$M = \left(\frac{\frac{1}{2}+1}{2}, \frac{\frac{1}{2}+1}{2}\right) = \left(\frac{3}{4}, \frac{3}{4}\right)$$

$$AM = \sqrt{\left(\frac{3}{4}-1\right)^2 + \left(\frac{3}{4}-1\right)^2} = \frac{\sqrt{2}}{2} \text{ و } AH = \frac{|1+1-1|}{\sqrt{1+1}} = \frac{1}{\sqrt{2}} \Rightarrow \frac{AM}{AH} = \frac{\frac{\sqrt{2}}{2}}{\frac{1}{\sqrt{2}}} = \frac{\sqrt{2}}{2} \times \sqrt{2} = 1$$

$m = -\frac{1}{r} \Rightarrow y = -\frac{1}{r}x + b \xrightarrow{y=0} y=b \Rightarrow B=(0, b)$
 $\xrightarrow{x=0} x=r b \Rightarrow A=(r b, 0)$
 از طرفی: $\frac{|0+0+b|}{\sqrt{(\frac{1}{r})^2+1}} = \sqrt{r^2} \Rightarrow |b| = r \Rightarrow b=r$
 $AB = \sqrt{10^2 + 5^2} = \sqrt{125} = 5\sqrt{5}$
 $B=(0, r)$
 $A=(10, r)$
 چون فاصله رو می‌خواهیم
 متغیر r مثبت بزرگتر از صفر
 نمی‌کند

$my - rx = r$ $\xrightarrow{y=0} x = -\frac{r}{r}$
 $y + m = x + r$ $\xrightarrow{y=0} x = \frac{r}{m-1}$
 $|A-B| = 9$
 C = محل تقاطع دو خط مستقیم
 CH = ارتفاع $\Rightarrow$ عرض نقطه C

$C = \text{محل تقاطع دو خط مستقیم}$
 $CH = \text{ارتفاع} \Rightarrow \text{عرض نقطه } C$
 $\frac{my - rx}{r} = \frac{y - r}{1 - m} \Rightarrow ry - 1 = (my - rx)(1 - m)$
 $ry - my + m^2y = 1 - rm + r^2m \Rightarrow y = \frac{r^2m - rm + 1}{m^2 - m + r} = \frac{r(m^2 - m + r)}{m^2 - m + r} = r$
 $\frac{rx(\frac{r}{m-1} + \frac{r}{r})}{r} = \frac{9}{r} \Rightarrow m^2 - m + r = 9m - 9$
 $m^2 - 4m + 9 = 0 \rightarrow P=9$

$B+B' = M \Rightarrow B = \frac{(r, r) + (x, y)}{2} \rightarrow B = (\frac{r+x}{2}, \frac{r+y}{2})$
 $-k - y_{B'} = r(k - x_{B'}) - 3 \Rightarrow y_{B'} = rx_{B'} - 9$

$m_{AA'} = -1 \rightarrow y = -x + b \rightarrow b = 3 \Rightarrow y = -x + 3$
 $A(1, 2)$
 $A'(-2, 3)$
 M = نقطه تقاطع
 $-x + 3 = x + 3 \Rightarrow x = 0$
 $0 + 3 = 0 + 3 \Rightarrow y = 3$
 $M(0, 3)$
 $\frac{A+A'}{2} = M \Rightarrow \frac{(1, 2) + (-2, 3)}{2} = (0, 3)$
 $A' = 2M - A = (0, 6) - (1, 2) = (-1, 4)$
 $rb - a = r(4) - (-3) = 15$