

$$A(4, -4) \quad B(2, 8)$$

$$m_{AB} = \frac{8 - (-4)}{2 - 4} = \frac{12}{-2} = -6 \rightarrow m'_{\text{عمود منتهی}} = \frac{-1}{-6} = \frac{1}{6}$$

$$M \left(\frac{4+2}{2}, \frac{8+(-4)}{2} \right) \rightarrow M(3, 2)$$

$$\frac{m' = \frac{1}{6}}{M(3, 2)} \rightarrow y - (2) = \frac{1}{6}(x - 3) \rightarrow \boxed{y = \frac{1}{6}x + \frac{5}{2}}$$

$$A(3, 1) \quad B(-4, 5) \quad C(-1, 3)$$

$$\overline{BC} = \sqrt{(-1 - (-4))^2 + (3 - 5)^2} = \sqrt{10} = \sqrt{2} \cdot \sqrt{5}$$

$$m_{AC} = \frac{3 - 1}{-1 - 3} = \frac{2}{-4} = -\frac{1}{2} \xrightarrow{(-1, 3)} y - 3 = -\frac{1}{2}(x + 1) \rightarrow y + \frac{1}{2}x - \frac{5}{2} = 0$$

$$\overline{AH} = \frac{|1 + \frac{1}{2} \cdot 3 - \frac{5}{2}|}{\sqrt{(\frac{1}{2})^2 + (-\frac{1}{2})^2}} = \frac{10}{\sqrt{2}} = 5\sqrt{2} \quad \overline{BC} - \overline{AH} = \sqrt{2} - 5\sqrt{2} = -4\sqrt{2}$$

$$2y - x = 5 \rightarrow y = \frac{1}{2}x + \frac{5}{2}$$

$$y = ax + 2 \rightarrow y = \frac{a}{2}x + 1$$

$$\text{داده شود} \rightarrow m = m' \rightarrow \frac{1}{2} = \frac{a}{2} \rightarrow a = 1$$

$$\Rightarrow r = \frac{|1 - 1 - (5)|}{\sqrt{1^2 + 1^2}} = \frac{4}{\sqrt{2}} \rightarrow r = \frac{2\sqrt{2}}{\sqrt{2}} = 2 \Rightarrow S = \pi r^2 = \pi \times (2)^2$$

$$\Rightarrow \boxed{S = 4\pi} \xrightarrow{\pi=3} \boxed{S = 12}$$

$$A(2, 3) \quad B(4, 1) \quad C(-1, -2)$$

$$M_{AB} \left(\frac{2+4}{2}, \frac{3+1}{2} \right) \rightarrow M(3, 2)$$

$$\overline{CM} = \sqrt{(3 - (-1))^2 + (2 - (-2))^2} = \sqrt{20} = 2\sqrt{5}$$

$$\overline{BC} = \sqrt{(4 - (-1))^2 + (1 - (-2))^2} = \sqrt{25} = 5$$

$$\Rightarrow \overline{BM} = \sqrt{(3 - 4)^2 + (2 - 1)^2} = \sqrt{2} \rightarrow \boxed{\overline{HM} = \frac{\sqrt{2}}{2}}$$

$$(3m - 2)x + (5 - m)y - 5 = 0 \rightarrow y = \left(\frac{2m - 3}{m - 5} \right)x + \frac{5}{m - 5}$$

$$(m + 1)x - (2m - 4)y + 1 = 0 \rightarrow y = \left(\frac{m + 1}{2m - 4} \right)x + \frac{1}{2m - 4}$$

$$\text{داده شود} \rightarrow \frac{m + 1}{2m - 4} = \frac{5 - m}{2m - 5} \rightarrow 3m^2 + m - 2 = -2m^2 + 13m - 10 \rightarrow 5m^2 - 12m + 8 = 0$$

$$\rightarrow \Delta < 0 \rightarrow \text{داده شده معادله} \rightarrow \boxed{\text{برای هر مقدار حقیقی m این خط به هم عمود نیستند}}$$

$$\begin{cases} AB: x+y=2 \rightarrow y = -\frac{1}{2}x + 2 \\ AC: y = x-1 \\ BC: x+y=4 \rightarrow y = -x+4 \end{cases}$$

$$\Rightarrow \overline{AH} = \frac{|1+1-4|}{\sqrt{1^2+1^2}} = \sqrt{2}$$

$$\overline{AM} = \sqrt{\left(\frac{2}{3}-1\right)^2 + \left(1-\frac{2}{3}\right)^2} = \frac{2}{3}\sqrt{2}$$

$$A: -\frac{1}{2}x + 2 = x-1 \rightarrow x=1, y=1 \rightarrow A(1,1)$$

$$B: -\frac{1}{2}x + 2 = -x+4 \rightarrow x=2, y=1 \rightarrow B(2,1)$$

$$C: x-1 = -x+4 \rightarrow x=\frac{5}{2}, y=\frac{3}{2} \rightarrow C\left(\frac{5}{2}, \frac{3}{2}\right)$$

$$\left. \begin{matrix} M\left(\frac{10}{3}, \frac{2}{3}\right) \\ BC \text{ bisecting } \end{matrix} \right\} \frac{\overline{AM}}{\overline{AH}} = \frac{\frac{2}{3}\sqrt{2}}{\sqrt{2}} = \frac{2}{3}$$

$$y = mx + b \quad m = -\frac{1}{2} \quad y = -\frac{1}{2}x + b \rightarrow y + \frac{1}{2}x - b = 0$$

$$\text{distance} \Rightarrow \frac{|10 + \frac{1}{2} \times 0 - b|}{\sqrt{1^2 + (\frac{1}{2})^2}} = \frac{|b|}{\frac{\sqrt{5}}{2}} = \sqrt{2} \rightarrow |b| = \sqrt{2}$$

$$\begin{matrix} \nearrow b = \sqrt{2} \rightarrow y = -\frac{1}{2}x + \sqrt{2} & \begin{cases} A: -\frac{1}{2}x + \sqrt{2} = 0 \rightarrow x=2\sqrt{2}, y=0 \\ B: y = \sqrt{2}, x=0 \end{cases} & AB = \sqrt{(2\sqrt{2})^2 + (\sqrt{2})^2} = \sqrt{10} \end{matrix}$$

$$\searrow b = -\sqrt{2} \rightarrow y = -\frac{1}{2}x - \sqrt{2} \quad \begin{cases} A': -\frac{1}{2}x - \sqrt{2} = 0 \rightarrow x=-2\sqrt{2}, y=0 \\ B': y = -\sqrt{2}, x=0 \end{cases} \quad A'B' = \sqrt{(-2\sqrt{2})^2 + (\sqrt{2})^2} = \sqrt{10}$$

$$my - fx = r \rightarrow y = \frac{f}{m}x + r \quad y=0 \rightarrow \frac{f}{m}x + r = 0 \rightarrow \frac{f}{m}x = -r \rightarrow x_1 = -\frac{m}{f}$$

$$y + mx = n + r \rightarrow y = (1-m)x + r \quad y=0 \rightarrow (1-m)x + r = 0 \rightarrow (1-m)x = -r \rightarrow x_2 = \frac{r}{1-m}$$

$$\rightarrow S = \frac{\Delta}{r} \quad \text{ارتفاع مثلث برابر دایره است} \rightarrow \text{مساحت مثلث} = \frac{1}{2} \times \text{طول وتر} \times \text{ارتفاع}$$

$$\Rightarrow S = \frac{\text{مساحت} \times \frac{r}{\Delta}}{r} = \frac{\Delta}{r} \rightarrow \text{مساحت} = \frac{\Delta}{r} = |m_1 - m_2| \quad \left(\frac{P}{m_1 - m_2} = -r \right)$$

$$\Rightarrow \left| \frac{r}{1-m} - \left(-\frac{m}{f}\right) \right| = \frac{\Delta}{r} \quad \begin{cases} \frac{r}{1-m} + \frac{m}{f} = \frac{\Delta}{r} \rightarrow r + m^2 - m = \Delta - \Delta m \rightarrow m^2 - 4m + 9 = 0 \rightarrow m = 3 \\ \frac{r}{1-m} + \frac{m}{f} = -\frac{\Delta}{r} \rightarrow r + m^2 - m = -\Delta - \Delta m \rightarrow m^2 + 6m - 1 = 0 \rightarrow P = -1 \end{cases}$$

$$y = 2x - 3$$

$$m_1 = 1 \rightarrow y_1 = 2 \times 1 - 3 = -1 \quad (1, -1) \xrightarrow{\text{نسبت}} m_1' = 2 \times 1 - 1 = 1, y_1' = 2 \times (-1) + 1 = -1$$

$$m_2 = 0 \rightarrow y_2 = 2 \times 0 - 3 = -3 \quad (0, -3) \xrightarrow{\text{نسبت}} m_2' = 2 \times 0 - 0 = 0, y_2' = 2 \times (-3) + 3 = -3$$

$$\rightarrow (+1, -1) \quad (+0, -3) \rightarrow m' = \frac{-1 - (-3)}{1 - 0} = 2$$

$$\Rightarrow y - (-1) = 2(x - 1) \rightarrow y = 2x - 1$$

$$y = x + \Delta \rightarrow m = 1 \rightarrow m' = -1 \xrightarrow{A(1,2)} y - 2 = -x + 1 \rightarrow y = -x + 3$$

$$\Rightarrow \text{مستقیم} \rightarrow x + \Delta = -x + 3 \rightarrow m = -1 \rightarrow M(-1, 4)$$

$$\begin{matrix} \text{نقطه } A(1,2) \text{ بر } \text{مستقیم} & \text{نقطه } A(1,2) \text{ بر } \text{مستقیم} & \begin{cases} m' = 2(-1) - 1 = -3 = a \\ y' = 2(4) - 2 = 6 = b \end{cases} \end{matrix}$$

$$\Rightarrow 2b - a = 2 \times 6 - (-3) = 15$$