

$A(4, -4) \quad B(2, 8)$

$m_{AB} = \frac{8 - (-4)}{2 - 4} = \frac{12}{-2} = -6 \rightarrow m'_{\text{عمود منتهی}} = \frac{-1}{-6} = \frac{1}{6}$

$M \left(\frac{4+2}{2}, \frac{1+(-4)}{2} \right) \rightarrow M(3, -1)$

$\frac{m' = \frac{1}{6}}{M(3, -1)} \rightarrow y - (-1) = \frac{1}{6}(x - 3) \rightarrow y = \frac{1}{6}x + \frac{5}{6}$

$A(3, 1) \quad B(-4, 5) \quad C(-1, 3)$

$\overline{BC} = \sqrt{(-1 - (-4))^2 + (3 - 5)^2} = \sqrt{10} = \sqrt{2} \cdot \sqrt{5}$

$m_{AC} = \frac{3 - 1}{-1 - 3} = \frac{2}{-4} = -\frac{1}{2} \xrightarrow{(-1, 3)} y - 3 = -\frac{1}{2}(x + 1) \rightarrow y + \frac{1}{2}x - \frac{5}{2} = 0$

$\overline{AH} = \frac{|1 + \frac{1}{2} \cdot 3 - \frac{5}{2}|}{\sqrt{(\frac{1}{2})^2 + (-\frac{1}{2})^2}} = \frac{1}{\frac{\sqrt{2}}{2}} = \sqrt{2} \quad \overline{BC} - \overline{AH} = \sqrt{2} - \sqrt{2} = 0$

$2y - x = 5 \rightarrow y = \frac{1}{2}x + \frac{5}{2}$

$fy = ax + 2 \rightarrow y = \frac{a}{f}x + \frac{2}{f}$

داده شود $\rightarrow m = m' \rightarrow \frac{1}{2} = \frac{a}{f} \rightarrow a = 2$

$\Rightarrow 2r = \frac{|1 - (-5)|}{\sqrt{2^2 + 1^2}} = \frac{6}{\sqrt{5}} \rightarrow r = \frac{3}{\sqrt{5}} \Rightarrow S = \pi r^2 = \pi \left(\frac{3}{\sqrt{5}} \right)^2$

$\Rightarrow S = \frac{9}{5}\pi \xrightarrow{\pi=3} S = \frac{27}{5}$

$A(2, 3) \quad B(4, 1) \quad C(-1, -2)$

$M_{AB} \left(\frac{2+4}{2}, \frac{3+1}{2} \right) \rightarrow M(3, 2)$

$\overline{CM} = \sqrt{(3 - (-1))^2 + (2 - (-2))^2} = \sqrt{20} = 2\sqrt{5}$

$\overline{BC} = \sqrt{(4 - (-1))^2 + (1 - (-2))^2} = \sqrt{25} = 5$

$\Rightarrow \overline{BM} = \sqrt{(3 - 4)^2 + (2 - 1)^2} = \sqrt{2} \rightarrow \overline{HM} = \frac{\sqrt{2}}{2}$

$(3m - 2)n + (5 - m)y - 5 = 0 \rightarrow y = \left(\frac{3m - 2}{m - 5} \right)n + \frac{5}{5 - m}$

$(m + 1)n - (3m - 4)y + 1 = 0 \rightarrow y = \left(\frac{m + 1}{3m - 4} \right)n + \frac{1}{3m - 4}$

داده شود $\rightarrow \frac{m + 1}{3m - 4} = \frac{5 - m}{3m - 4} \rightarrow 3m^2 + m - 2 = -3m^2 + 13m - 20 \rightarrow 6m^2 - 12m + 18 = 0$

$\rightarrow \Delta < 0 \rightarrow \text{دو جوابی نیست} \rightarrow \text{برای هر مقدار مقبوعی } m \text{ این دو خط هم‌خط نیستند}$

$$\begin{cases} AB: x+y=2 \rightarrow y = -\frac{1}{2}x + 2 \\ AC: y = x-1 \\ BC: x+y=4 \rightarrow y = -x+4 \end{cases}$$

$$\Rightarrow \overline{AH} = \frac{|1+1-4|}{\sqrt{1^2+1^2}} = \sqrt{2}$$

$$\overline{AM} = \sqrt{\left(\frac{2}{3}-1\right)^2 + \left(1-\frac{1}{3}\right)^2} = \frac{2}{3}\sqrt{2}$$

$$A: -\frac{1}{2}x + 2 = x-1 \rightarrow x=1, y=1 \rightarrow A(1,1)$$

$$B: -\frac{1}{2}x + 2 = -x+4 \rightarrow x=2, y=1 \rightarrow B(2,1)$$

$$C: x-1 = -x+4 \rightarrow x=\frac{5}{2}, y=\frac{3}{2} \rightarrow C\left(\frac{5}{2}, \frac{3}{2}\right)$$

$$\left. \begin{matrix} M\left(\frac{10}{3}, \frac{4}{3}\right) \\ B(2,1) \end{matrix} \right\} \frac{\overline{AM}}{\overline{AH}} = \frac{\frac{2}{3}\sqrt{2}}{\sqrt{2}} = \frac{2}{3}$$

$$y = mx + b \quad m = -\frac{1}{2} \quad y = -\frac{1}{2}x + b \rightarrow y + \frac{1}{2}x - b = 0$$

$$\text{المسافة} \Rightarrow \frac{|10 + \frac{1}{2} \times 0 - b|}{\sqrt{1^2 + (\frac{1}{2})^2}} = \frac{|b|}{\frac{\sqrt{5}}{2}} = \sqrt{20} \rightarrow |b| = 20$$

$$\begin{matrix} \nearrow b=20 \rightarrow y = -\frac{1}{2}x + 20 \\ \searrow b=-20 \rightarrow y = -\frac{1}{2}x - 20 \end{matrix} \quad \begin{matrix} A: -\frac{1}{2}x + 20 = 0 \rightarrow x=40, y=0 \\ B: y=20, x=0 \end{matrix} \quad \left. \begin{matrix} AB = \sqrt{(40)^2 + 20^2} = 40\sqrt{5} \\ A'B' = \sqrt{(-40)^2 + (-20)^2} = 40\sqrt{5} \end{matrix} \right\}$$

$$\begin{matrix} \nearrow b=20 \rightarrow y = -\frac{1}{2}x + 20 \\ \searrow b=-20 \rightarrow y = -\frac{1}{2}x - 20 \end{matrix} \quad \begin{matrix} A': -\frac{1}{2}x - 20 = 0 \rightarrow x=-40, y=0 \\ B': y=-20, x=0 \end{matrix} \quad \left. \begin{matrix} A'B' = \sqrt{(-40)^2 + (-20)^2} = 40\sqrt{5} \end{matrix} \right\}$$

$$my - fx = r \rightarrow y = \frac{f}{m}x + r \quad y=0 \rightarrow \frac{f}{m}x + r = 0 \rightarrow \frac{f}{m}x = -r \rightarrow x_1 = -\frac{m}{f}$$

$$y + mx = n + r \rightarrow y = (-m)x + r \quad y=0 \rightarrow (-m)x + r = 0 \rightarrow (-m)x = -r \rightarrow x_2 = \frac{r}{m-1}$$

$$\rightarrow S = \frac{\Delta}{r} \quad \text{ارتفاع مثلث بارتفاعه} \rightarrow \text{مساحة مثلث} = \frac{1}{2} \times \text{القاعدة} \times \text{الارتفاع}$$

$$\Rightarrow S = \frac{\text{مساحة} \times \frac{r}{\Delta}}{r} = \frac{\Delta}{r} \rightarrow \text{مساحة} = \frac{\Delta}{r} = |m_1 - m_2| \quad \left(\frac{P}{m_1 - m_2} = -r \right)$$

$$\Rightarrow \left| \frac{r}{m-1} - \left(-\frac{m}{f}\right) \right| = \frac{\Delta}{r} \quad \begin{cases} \frac{r}{m-1} + \frac{m}{f} = \frac{\Delta}{r} \rightarrow r + m^2 - m = \Delta m - \Delta \rightarrow m^2 - 4m + 9 = 0 \rightarrow m=3 \\ \frac{r}{m-1} + \frac{m}{f} = -\frac{\Delta}{r} \rightarrow r + m^2 - m = -\Delta m - \Delta \rightarrow m^2 + 5m - 1 = 0 \rightarrow P = -1 \end{cases}$$

$$y = 2x - 3$$

$$m_1 = 1 \rightarrow y_1 = 2 \times 1 - 3 = -1 \quad (1, -1) \xrightarrow{\text{نقطة نسب}} m_1' = 2 \times 1 - 1 = 1, y_1' = 2 \times (-1) + 1 = -1$$

$$m_2 = 0 \rightarrow y_2 = 2 \times 0 - 3 = -3 \quad (0, -3) \xrightarrow{M(2, -2)} m_2' = 2 \times 0 - 0 = 0, y_2' = 2 \times (-2) + 3 = -1$$

$$\rightarrow (+2, -1) \quad (+2, -1) \rightarrow m' = \frac{-1 - (-3)}{2 - 0} = 1$$

$$\Rightarrow y - (-1) = 1(x - 2) \rightarrow y = x - 3$$

$$y = x + 2 \rightarrow m = 1 \rightarrow m' = -1 \quad A(1, 2) \rightarrow y - 2 = -x + 1 \rightarrow y = -x + 3$$

$$\Rightarrow \text{مسافة} \rightarrow x + 2 = -x + 3 \rightarrow x = \frac{1}{2} \rightarrow M(-\frac{1}{2}, 4)$$

$$\begin{matrix} \text{نقطة} A(1, 2) \rightarrow \text{نقطة} A(1, 2) \\ y = x + 2 \quad M(-\frac{1}{2}, 4) \end{matrix} \rightarrow \begin{matrix} m' = 2(-1) - 1 = -3 = a \\ y' = 2(4) - 2 = 6 = b \end{matrix}$$

$$\Rightarrow 2b - a = 2 \times 6 - (-3) = 15$$