

کوتاهترین مسافت (مسئله ۱)

مسئله (۲)

مسئله (۳)

$$m_{AB} = \frac{\Delta y}{\Delta x} = \frac{1-4}{-2-1} = -\frac{3}{3} = -1$$

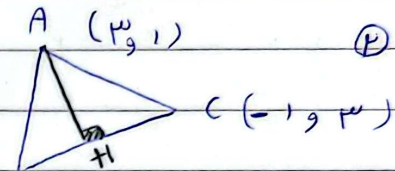
① معادله خط از دو نقطه $A(1, 4)$ و $B(2, 1)$ را بنویسید

نقطه $A(1, 4)$

معادله خط: $y = -\frac{1}{3}x + b$ از $(1, 4)$ داریم: $4 = -\frac{1}{3} + b \rightarrow b = \frac{13}{3}$

$BC = \sqrt{14 + 9} = 5$

$m_{BC} = \frac{3}{-2} = -\frac{3}{2}$ معادله خط: $y = -\frac{3}{2}x + b$ از $(-1, 3)$ داریم: $3 = \frac{3}{2} + b \rightarrow b = \frac{3}{2}$



معادله خط BC: $3y + 2x - 3 = 0$

فاصله $AH = \frac{|3(1) + 2(4) - 3|}{\sqrt{2^2 + 3^2}} = \frac{10}{5} = 2$

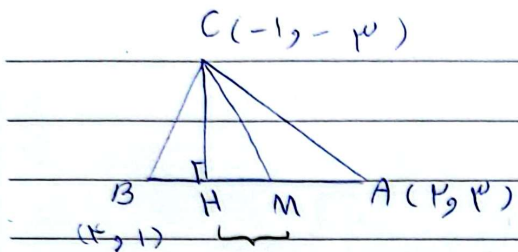
$BC - AH = 5 - 2 = 3$

مسئله (۴) معادله خط: $2y - x = 5$ یا $y = \frac{1}{2}x + \frac{5}{2}$

فاصله از مبدأ تا خط: $\frac{|2(0) - 1(0) - 5|}{\sqrt{2^2 + (-1)^2}} = \frac{5}{\sqrt{5}} = \sqrt{5}$

مساحت مثلث: $S = \frac{1}{2} \times \text{طول وتر} \times \text{فاصله از مبدأ} = \frac{1}{2} \times \sqrt{5} \times \sqrt{5} = \frac{5}{2}$

s.a.m



$$MH = \sqrt{\left(\frac{1}{p}\right)^2 + \left(\frac{1}{p}\right)^2} = \sqrt{\frac{2}{p^2}}$$

$$M = AB \text{ midpoint} = (p, p)$$

$$AB \Rightarrow y = -x + \omega \quad y + x - \omega = 0$$

$$mCH = 1 \quad CH: y = x - p$$

$$H? \rightarrow x - p = -x + \omega \rightarrow 2x = \omega + p \rightarrow x = \frac{\omega + p}{2}$$

$$y = \frac{p}{p} \quad H\left(\frac{\omega + p}{2}, \frac{p}{p}\right)$$

$$\begin{cases} (p-m-p)x + (\omega-m)y - \omega = 0 \\ (m+1)x - (pm-\varepsilon)y + 1 = 0 \end{cases}$$

$$\frac{m+1}{pm-\varepsilon} = \frac{m-\omega}{p-3m} \rightarrow \omega m p - 1pm + 1\omega = 0 \quad \Delta < 0 \quad m \text{ is not real}$$

$$\begin{cases} AB: y = -\frac{1}{p}x + \frac{p}{p} \\ AC: y = px - 1 \\ BC: y = -x + \varepsilon \end{cases}$$

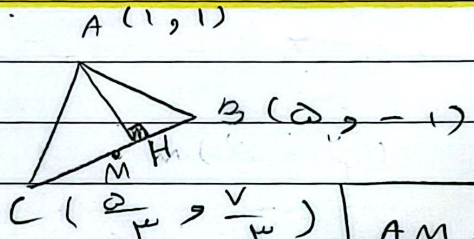
$$y + x - \varepsilon = 0$$

$$A: -\frac{1}{p}x + \frac{p}{p} = px - 1 \rightarrow A(1, 1)$$

$$B: -\frac{1}{p}x + \frac{p}{p} = -x + \varepsilon \rightarrow B(\omega, -1)$$

$$C: px - 1 = -x + \varepsilon \rightarrow C\left(\frac{\omega + p}{p}, \frac{p}{p}\right)$$

$$BC \text{ midpoint} = \left(\frac{1+p}{p}, \frac{p}{p}\right)$$



$$AM = \sqrt{\left(\frac{\omega + p}{p}\right)^2 + \left(\frac{1}{p}\right)^2} = \frac{\omega \sqrt{p}}{p}$$

$$AH = \frac{|1 + 1 - \varepsilon|}{\sqrt{1+1}} = \sqrt{p}$$

$$\frac{\frac{\omega \sqrt{p}}{p}}{\sqrt{p}} = \frac{\omega}{p}$$

s.a.m

$$y = -\frac{1}{r}x + p \rightarrow 2y + x - pb = 0 \quad \text{⑤}$$

$$|b| = \frac{|pb|}{\sqrt{r^2 + 1}} = \frac{|rpb|}{\sqrt{r^2 + 1}} = r\sqrt{r^2 + 1} \quad |b| = a$$

$$y = -\frac{1}{r}x \pm a \rightarrow \text{مركز دایره} = 10 \quad \begin{cases} A(10, 0) \\ B(0, a) \end{cases}$$

$$AB = \sqrt{10^2 + r^2 a^2} = \sqrt{100 + r^2 a^2} = \boxed{a\sqrt{r^2 + 1}}$$

$$\begin{cases} y = (1-m)x + r = 0 \rightarrow x = \frac{r}{m-1} \\ y = \frac{r}{m}x + r = 0 \rightarrow x = -\frac{m}{r} \end{cases} \quad \text{فاصله بین دو نقطه} = \frac{r}{m-1} \times \frac{r}{m} = \frac{r^2}{m(m-1)} \quad \text{⑥}$$

$$(1-m)x + r = \frac{r}{m}x + r \rightarrow x = 0 \quad y = r \quad \text{دایره به خط مماس در } (0, r)$$

$$\frac{1}{r} \times \frac{r^2}{m(m-1)} \times r = \frac{a}{r} \quad \frac{r^2}{m(m-1)} = \frac{a}{r} \quad \frac{r^2}{m-1} + \frac{m^{(m-1)}}{r} = \frac{r^2 + m^2 - m}{r(m-1)}$$

$$\frac{m^2 - m + r^2}{r(m-1)} = \frac{a}{r} \quad m^2 - m + r^2 = am - a \rightarrow m = r$$

$$m^2 - 4m + 4 = 0 \quad (m-2)^2 = 0$$

$$\begin{aligned} x \rightarrow r(r) - x &= \varepsilon - x & \text{فاصله بین } P(r, -r) \text{ و } y = rx - r & \text{ ⑦} \\ y \rightarrow r(-r) - y &= -r - y & -r - y &= r(r - x) - r \\ & & -\varepsilon - y &= a - rx \end{aligned}$$

$$\boxed{y = rx - a}$$

s.a.m

$$y - a - a = 0$$

$$y = x + a = AA' \text{ (inverse)} \rightarrow m AA' = -1$$

$$A(1, r) \rightarrow A'(a, b) \quad \frac{b-r}{a-1} = -1 \quad b-r = 1-a \quad b = r-a \quad (10)$$

$$|r - a| = |b - a - a| \quad |b - a - a| = r$$

$$\sqrt{1+r} \quad \sqrt{1+r}$$

$$\begin{cases} b-a-a = r \rightarrow b = a+a = r-a \rightarrow \begin{matrix} a = -r \\ b = r \end{matrix} \quad (1) \end{cases}$$

$$\begin{cases} b-a-a = -r \rightarrow b = a+1 = r-a \rightarrow \begin{matrix} a = 1 \\ b = r \end{matrix} \quad (2) \end{cases}$$

$$(1) \quad r-b-a = r+r = (1+r) / (2) \quad r-b-a = r-1 = (r)$$

s.a.m