باسمه تعالی

مریم جعفری

① $A(۴,-۶)$ $B(۲,۸)$

وسط نقطه $A, B \Rightarrow x = \frac{x_A + x_B}{۲} = \frac{۲+۴}{۲} = ۳$

$y = \frac{y_A + y_B}{۲} = \frac{۸-۶}{۲} = ۱$ $(۳,۱)$

$m_{AB} = \frac{\Delta y}{\Delta x} = \frac{۸-(-۶)}{۲-۴} = \frac{۱۴}{-۲} = -۷$

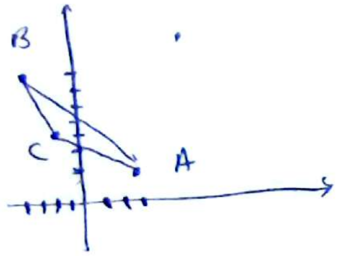
$y = mx + b \rightarrow y = -۷x + b \xrightarrow{(۲,۸)} ۸ = -۷ \times ۲ + b \Rightarrow b = ۲۲$

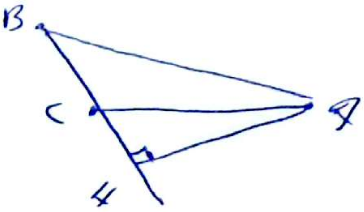
$y = -۷x + ۲۲$

$m_{AB} \times m_{\text{عمود}} = -۱ \Rightarrow -۷ \times m_{\text{عمود}} = -۱ \Rightarrow \boxed{m_{\text{عمود}} = \frac{۱}{۷}}$

$y = \frac{۱}{۷}x + b \xrightarrow{(۳,۱)} ۱ = \frac{۳}{۷} + b \Rightarrow b = \frac{۴}{۷} \Rightarrow y = \frac{۱}{۷}x + \frac{۴}{۷}$

②





$A(۳,۱)$
$B(-۴,۷)$
$C(-۱,۳)$

فاصله B از C = طول ضلع BC $\Rightarrow \sqrt{(x_B - x_C)^۲ + (y_B - y_C)^۲} = \sqrt{(-۴-(-۱))^۲ + (۷-۳)^۲} = ⑤$

فاصله نقطه A از خط BC

$m_{BC} = \frac{\Delta y}{\Delta x} = \frac{۷-۳}{-۴-(-۱)} = \frac{۴}{-۳} = -\frac{۴}{۳}$

$y = mx + b$

$y = -\frac{۴}{۳}x + b \xrightarrow{(-۱,۳)} ۳ = -\frac{۴}{۳} \times -۱ + b \Rightarrow b = \frac{۵}{۳} \Rightarrow \boxed{y = -\frac{۴}{۳}x + \frac{۵}{۳}}$

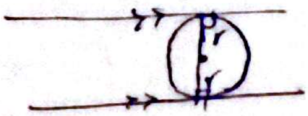
$\xrightarrow{\times ۳} ۳y + ۴x - ۵ = ۰$

$\frac{|ax + by + c|}{\sqrt{a^۲ + b^۲}} = \frac{|y + \frac{۴}{۳}x - \frac{۵}{۳}|}{\sqrt{۱ + (\frac{۴}{۳})^۲}} = \frac{|۳y + ۴x - ۵|}{\sqrt{۳^۲ + ۴^۲}} = \frac{|۳ \times ۱ + ۴ \times ۳ - ۵|}{۵}$

$= \frac{|۳ + ۱۲ - ۵|}{۵} = \frac{۱۰}{۵} = \boxed{۲}$

$BC - AH = ۵ - ۲ = ۳$

③



$۲r$ = فاصله دو خط

$۲y - x - ۷ = ۰ \Rightarrow$ ④

$۴y - ax - ۴ = ۰$

$(۲y - x - ۷ = ۰) \times ۲ \Rightarrow ۴y - ۲x - ۱۴ = ۰$

خط موازی غیر منطبق $\frac{a}{a'} = \frac{b}{b'} \neq \frac{c}{c'} = \frac{۲}{۴} = \frac{-۱}{-a} \neq \frac{-۷}{-۴}$

$-۲a = -۴ \Rightarrow \boxed{a = ۲}$

بنام خداوند دانش

$(2y - x - 7 = 0) \times 2 = 4y - 2x - 14 = 0 \quad 4y - 2x = 14$

$4y - 2x - 4 = 0 \Rightarrow 4y - 2x = 4$

$$\frac{|c-c'|}{\sqrt{a^2+b^2}} = \frac{|14-4|}{\sqrt{4^2+2^2}} = \frac{10}{\sqrt{20}} = \frac{2\times 5}{2\sqrt{5}} = \sqrt{5} = 2r$$

$$r = \frac{\sqrt{5}}{2} \qquad S = \pi r^2 = \pi \times \left(\frac{\sqrt{5}}{2}\right)^2 = \boxed{\frac{5}{4}\pi}$$

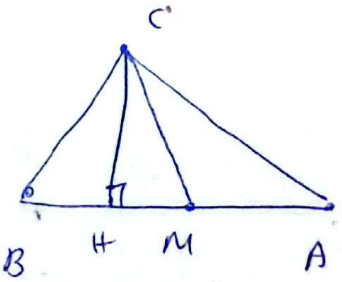
(4)

$A(2,3)$

$B(4,1)$

$C(-1,-3)$

$MH = ?$



$$M \Rightarrow x_M = \frac{x_A + x_B}{2} = \frac{2+4}{2} = 3 \qquad M(3,2)$$
$$y_M = \frac{y_A + y_B}{2} = \frac{3+1}{2} = 2$$

$m_{AB} = \frac{\Delta y}{\Delta x} = \frac{3-1}{2-4} = \frac{2}{-2} = -1$ خط $AB \Rightarrow y = ax + b$

$\underbrace{m_{AB}}_{-1} \times m_{CH} = -1 \Rightarrow m_{CH} = 1$ $\Rightarrow y = -x + b \xrightarrow{A(2,3)} 3 = -2 + b$ $\boxed{b = 5}$

خط $CH \to y = ax + b \to y = x + b \xrightarrow{(-1,-3)} -1 + b = -3 \Rightarrow \boxed{b = -2}$ $\boxed{y = -x + 5}$

$\to \boxed{y = x - 2}$

محل برخورد $CH, AB \Rightarrow \begin{cases} y = x - 2 \\ y = -x + 5 \end{cases} \Rightarrow x - 2 = -x + 5 \quad H\left(\frac{7}{2}, \frac{3}{2}\right)$

$2x = 7 \Rightarrow x = \frac{7}{2}$

$y = x - 2 \Rightarrow y = \frac{7}{2} - 2 = \boxed{\frac{3}{2}}$

$$MH = \sqrt{(x_M - x_H)^2 + (y_M - y_H)^2}$$

$$= \sqrt{\underbrace{\left(3 - \frac{7}{2}\right)^2}_{\left(-\frac{1}{2}\right)^2} + \underbrace{\left(2 - \frac{3}{2}\right)^2}_{\left(\frac{1}{2}\right)^2}} = \sqrt{\frac{2}{4}} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \boxed{\frac{\sqrt{2}}{2}}$$

$\frac{7}{2} - \frac{4}{2} = \boxed{\frac{3}{2}}$

۵) $m = ?$

خط $A \Rightarrow (3m-2)x + (5-m)y - 7 = 0 , \quad m = -\frac{\text{ضریب } x}{\text{ضریب } y} = \frac{-(3m-2)}{5-m} = \frac{-3m+2}{5-m}$

خط $B \Rightarrow (m+1)x - (2m-4)y + 1 = 0 , \quad m = \frac{-\text{ضریب } x}{\text{ضریب } y} = \frac{-(m+1)}{-(2m-4)} = \frac{m+1}{2m-4}$

بر هم عمودند

$m_{A \text{ خط}} \times m_{B \text{ خط}} = -1 \Rightarrow \frac{-3m+2}{5-m} \times \frac{m+1}{2m-4} = -1 \Rightarrow \frac{-3m+2}{5-m} = \frac{-2m+4}{m+1}$

$(5-m)(-2m+4) = (-3m+2)(m+1)$

$\Rightarrow -10m + 20 + 2m^2 - 4m = -3m^2 - 3m + 2m + 2$

ریشه ندارد

$2m^2 - 14m + 20 = -3m^2 - m + 2 \qquad \Delta < 0 \quad b^2 - 4ac = (-13)^2 - 4 \times 5 \times 18$

$5m^2 - 13m + 18 = 0 \longrightarrow (\div 5) \quad m^2 - 13m + (18 \times 5) = 0 \qquad 169 - 360 = -191$

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۲) $\frac{AM}{AH}$

$AB: x + 2y = 3$

$AC: y = 2x - 1$

$BC: x + y = 4$

تقاطع $= A \Rightarrow (1,1) \quad x + 2y = 3 \Rightarrow 2y = 3 - x \Rightarrow y = \frac{3-x}{2}$

$y = 2x - 1$

$\frac{3-x}{2} = 2x - 1 \Rightarrow 3 - x = 4x - 2$

$5x = 5 \Rightarrow x = 1$

$y = 2x - 1 \longrightarrow 2 \times 1 - 1 = 1$

تقاطع

$B \Rightarrow (5,-1) \quad x + 2y = 3 \Rightarrow 2y = 3 - x \Rightarrow y = \frac{3-x}{2}$

$x + y = 4 \Rightarrow y = 4 - x \quad \Rightarrow \frac{3-x}{2} = 4 - x \Rightarrow$

$3 - x = 8 - 2x \Rightarrow x = 5$

$y = 4 - x \Rightarrow y = 4 - 5 = -1$

تقاطع

$C \Rightarrow (\frac{5}{3}, \frac{7}{3}) \quad y = 2x - 1$

$x + y = 4 \Rightarrow y = 4 - x$

$2x - 1 = 4 - x \Rightarrow 3x = 5 \Rightarrow x = \frac{5}{3}$

$y = 2x - 1 \to y = 2 \times \frac{5}{3} - 1 = \frac{10}{3} - \frac{3}{3} = \frac{7}{3}$

$y_M = \frac{y_B + y_C}{2} = \frac{-1 + \frac{7}{3}}{2} = \frac{\frac{4}{3}}{2} = \boxed{\frac{2}{3}}$

$M \Rightarrow x_M = \frac{x_B + x_C}{2} = \frac{5 + \frac{5}{3}}{2} = \frac{\frac{20}{3}}{2} = \boxed{\frac{10}{3}} \qquad M (\frac{10}{3}, \frac{2}{3})$

$AM = \sqrt{(x_A - x_M)^2 + (y_A - y_M)^2} = \sqrt{(1 - \frac{10}{3})^2 + (1 - \frac{2}{3})^2} = \sqrt{\frac{50}{9}} = \frac{\sqrt{50}}{3} = \boxed{\frac{5\sqrt{2}}{3}}$

$(-\frac{7}{3})^2 = \frac{49}{9} \qquad (\frac{1}{3})^2 = \frac{1}{9}$

$y = ax + b$

$m_{BC} \Rightarrow y = 4 - x \Rightarrow \boxed{m_{BC} = -1}$

$m_{AH} \times m_{BC} = -1 \Rightarrow \boxed{m_{AH} = 1}$

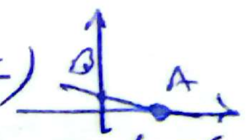
$y = ax + b \Rightarrow y = x + b \xrightarrow{(1,1)} y = 1 + b = 1 \Rightarrow b = 0 \Rightarrow y = x$ خط AH

فاصله A از خط BC $= \frac{|ax + by + c|}{\sqrt{a^2 + b^2}} = \frac{|x + y - 4|}{\sqrt{1^2 + 1^2}} = \frac{|1 + 1 - 4|}{\sqrt{2}} = \frac{2}{\sqrt{2}}$

$\frac{AM}{AH} = \frac{\frac{5\sqrt{2}}{3}}{\frac{2}{\sqrt{2}}} = \frac{5 \times 2}{2 \times 3} = \boxed{\frac{5}{3}}$

به نام خداوند دانش

(۷)



$(0, -\frac{c}{b})$, $(-\frac{c}{a}, 0)$

$$\text{فاصله‌ی نقطه‌ی } (0,0) \text{ از خط} = \frac{|ax+by+c|}{\sqrt{a^2+b^2}} = \frac{|0\times a + 0\times b + c|}{\sqrt{a^2+b^2}} = \sqrt{20}$$

$m = -\frac{1}{2}$

فاصله‌ی مبدأ تا خط $= \sqrt{20}$

محور مختصات در نقطه همان B، A

فاصله‌ی نقطه A و B از هم

$$\Rightarrow \frac{|c|}{\sqrt{a^2+b^2}} = \sqrt{20} \Rightarrow \frac{|c|}{\sqrt{a^2+(2a)^2}} = \frac{|c|}{\sqrt{5a^2}} = \sqrt{20} = 2\sqrt{5}$$

$2a = b$, $(2a)^2 = 4a^2$

$$m = -\frac{\text{ضریب } x}{\text{ضریب } y} = \frac{-a}{b} = -\frac{1}{2} \Rightarrow \boxed{2a = b}$$

$$\Rightarrow \frac{|c|}{\sqrt{5}\,|a|} = 2\sqrt{5} \Rightarrow \boxed{\frac{|c|}{|a|} = 10}$$

$$\text{فاصله‌ی نقطه B، A} = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2} = \sqrt{(0-(-\frac{c}{a}))^2 + (-\frac{c}{b} - 0)^2} = \sqrt{\frac{5c^2}{4a^2}} = \frac{\sqrt{5}\,|c|}{2\,|a|}$$

$\frac{c^2}{a^2} = \frac{4c^2}{4a^2}$, $\frac{c^2}{b^2} = \frac{c^2}{(2a)^2} = \frac{c^2}{4a^2}$

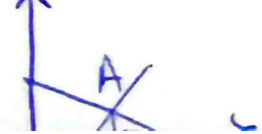
$$= \frac{\sqrt{5}}{2} \times 10 = \boxed{5\sqrt{5}}$$

(۸)

مساحت مثلث بین خطوط

$\frac{5}{2}$

(A) (B)

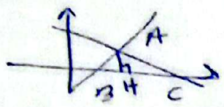


به نام خداوند دانشمند

۸) مساحت مثلث بین خطوط $= \frac{5}{2}$ ، رسم خط ، از کدام نقطه خاص می‌گذرد

(A) $my - 4x = 2m$ ، محور x ها

(B) $y + mx = x + 2$ ، $y = 0$



محل برخورد خط (A) و محور x: $\begin{cases} my - 4x = 2m \\ y = 0 \end{cases} \Rightarrow -4x = 2m \Rightarrow \boxed{x = -\frac{1}{2}m}$

محل برخورد خط (B) و محور x: $\begin{cases} y + mx = x + 2 \\ y = 0 \end{cases} \Rightarrow mx = x + 2 \Rightarrow \boxed{x = \frac{2}{m-1}}$

$my - 4x = 2m$ ، $\begin{cases} m = 1 & y - 4x = 2 \ (B) \\ m = 2 & 2y - 4x = 4 \ (A) \end{cases} \Rightarrow A - B \Rightarrow \boxed{y = 2}$ ، $(0,2)$

$y - 4x = 2 \Rightarrow -4x = 0 \Rightarrow \boxed{x = 0}$

$y + mx = x + 2$ ، $\begin{cases} m = 1 & y + x = x + 2 \Rightarrow \boxed{y = 2} \\ m = 0 \Rightarrow & y = x + 2 \end{cases} \Rightarrow y = x + 2$ ، $(0,2)$

$2 = x + 2 \Rightarrow \boxed{x = 0}$

$A(0,2)$ ، $(0,2)$ است ؛ هر دو دسته خط از یک نقطه می‌گذرند $\Rightarrow$ محل برخورد این دو خط

فاصله‌ی نقطه C و B $= \sqrt{(x_B - x_C)^2 + \underbrace{(y_B - y_C)^2}_{(0-0)^2}} = \sqrt{\left(-\frac{m}{2} - \frac{2}{m-1}\right)^2}$

فاصله‌ی A از خط BC $\Rightarrow y_A = 2 = AH$

$= \left| \frac{-m^2 + m - 4}{2m - 2} \right| = \frac{5}{2}$

$\frac{1}{2} AH \times BC = \frac{5}{2} \Rightarrow 2 \times BC = 5 \Rightarrow BC = \frac{5}{2}$

$\longrightarrow \frac{-m^2 + m - 4}{2m - 2} = \frac{5}{2} \Rightarrow -2m^2 + 2m - 8 = 10m - 10$

$-2m^2 - 8m + 2 = 0 \xrightarrow{\times -\frac{1}{2}} m^2 + 4m - 1 = 0$

$\sqrt{20} = 2\sqrt{5}$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{16 - 4 \times 1 \times -1}}{2} = \boxed{-2 \pm \sqrt{5}}$

$\frac{-m^2 + m - 4}{2m - 2} = \frac{-5}{2} \Rightarrow$

$-2m^2 + 2m - 8 = -10m + 10 \Rightarrow -2m^2 + 12m - 18 = 0$

$\xrightarrow{\div -2} m^2 - 6m + 9 = 0 \longrightarrow (m-3)^2 = 0 \Rightarrow \boxed{m = 3}$

$\underbrace{(-2 + \sqrt{5})(-2 - \sqrt{5})}_{4 - 5 = -1}(3) = \boxed{-3}$

به نام خداوند دانا

⑨ قرینه خط $y=2x-3$ نسبت به نقطه $M(2,-2)$

نقطه $A(2,1)$ به دلخواه از این خط در نظر می‌گیریم $y=2\times2-3=1$

$m=2$ ⇐ موازی ⇐ شیب‌ها مساوی

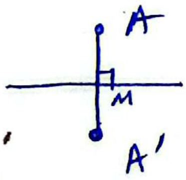
$A'=?\quad x_{A'}=2x_M-x_A=2\times2-2=2\qquad(2,-5)$

$y_{A'}=2y_M-y_A=\underbrace{2\times-2}_{-4}-1=-5$

$y=2x+b \xrightarrow{A'(2,-5)} -5=2\times2+b \Rightarrow$ $\boxed{b=-9}$ $\boxed{y=2x-9}$

⑩ $A(1,2)\xrightarrow{y=x+5}A'(a,b)$

$2b-a$



⇐ شیب AA' ⇐ شیب $y\times$ شیب $AA'=-1$

$1\times\bigcirc=-1\Rightarrow$ $\boxed{\text{شیب } AA'=-1}$

$y=ax+b\Rightarrow y=-x+b$

$\xrightarrow{A(1,2)} 2=-1+b\Rightarrow\boxed{b=3}$

$\Rightarrow\boxed{y=-x+3}$

محل برخورد دو خط M $\Rightarrow\begin{cases}y=-x+3\\y=x+5\end{cases}$

$-x+3=x+5$

$2x=-2\Rightarrow x=-1\Rightarrow y=-x+3\Rightarrow\boxed{y=+1+3=4}$

$(-1,4)$

$2x_M-x_A=x_{A'}\Rightarrow\underbrace{2\times-1}_{-2}-1=-3$

$(-3,6)$

$2y_M-y_A=y_{A'}\Rightarrow\underbrace{2\times4}_{8}-2=6$

$A'(-3,6)\Rightarrow 2b-a=\underbrace{2\times6}_{12}-(-3)_{+3}=\boxed{15}$