الیسا جلالی - تکلیف شماره ۱۶

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$m = \frac{8-(-6)}{2-4} = -7 \longrightarrow m' = \frac{1}{7}$

$M \left| \begin{array}{l} \frac{4+2}{2} = 3 \\ \frac{8-6}{2} = 1 \end{array} \right.$

$\frac{1}{7}(x-3) = y-1 \qquad \frac{1}{7}x - \frac{3}{7} + 1 = y$

$y = \frac{1}{7}x + \frac{4}{7}$

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$|BC| = \sqrt{(7-3)^2 + (-4-(-1))^2} = \sqrt{16+9} = 5$

$m_{BC} = \frac{7-3}{-4-(-1)} = -\frac{4}{3} \qquad BC \to -\frac{4}{3}(x-1) = y-3 \qquad y = -\frac{4}{3}x + \frac{5}{3}$

$y + \frac{4}{3}x - \frac{5}{3} = 0$

فاصله A از BC $= AH = \frac{|1+4-\frac{5}{3}|}{\sqrt{1+\frac{16}{9}}} = \frac{\frac{10}{3}}{\frac{5}{3}} = 2$

$BC - AH = 5 - 2 = 3$

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قطر دایره $= \frac{12}{\sqrt{16+4}} = \frac{12}{2\sqrt{5}} = \frac{6}{\sqrt{5}} \xrightarrow{\div 2} r = \frac{3}{\sqrt{5}} \to S = \pi r^2 = \pi \times \frac{9}{5} = 1.8\pi$

$\begin{cases} y = ax + 2 \\ 2y = x + 7 \xrightarrow{\times 2} 4y = 2x + 14 \end{cases} \to a = 2 \qquad \begin{cases} y - 2x = 2 \\ 4y - 2x = 14 \end{cases}$

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$M \left| \begin{array}{l} 3 \\ 2 \end{array} \right. \qquad m_{AB} = \frac{3-1}{2-4} = -1 \qquad AB: -(x-2) = y-3$

$y = -x + 5$

$m_{CH} = 1 \to CH: x+1 = y+3 \qquad y = x - 2$

$2x = 7 \quad x = \frac{7}{2} \qquad y = \frac{3}{2}$

$H \left| \begin{array}{l} \frac{7}{2} \\ \frac{3}{2} \end{array} \right.$

$|MH| = \sqrt{(\frac{7}{2}-3)^2 + (\frac{3}{2}-2)^2} = \sqrt{\frac{1}{4}+\frac{1}{4}} = \frac{\sqrt{2}}{2}$

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$ax + by + c = 0 \xrightarrow{\text{دو خط عمود}} aa' + bb' = 0$

$(m+1)(3m-2) + (4-2m)(5-m) = 0$

$3m^2 + m - 2 + 20 - 14m + 2m^2 = 0 \qquad 5m^2 - 13m + 18 = 0 \to$ $\Delta < 0$ پس به ازای هیچ مقدار m

$A: \quad \frac{3-x}{2} = 2x-1 \qquad 3-x = 4x-2 \qquad 5x = 5 \qquad x=1,\ y=1$

$B: \quad 3-2y = 4-y \qquad y=-1,\ x=5$

$C: \quad 2x-1 = 4-x \qquad 3x = 5 \qquad x = \frac{5}{3}, \quad y = \frac{7}{3}$

$M \left| \begin{array}{l} \frac{10}{3} \\ \frac{2}{3} \end{array} \right. \qquad |AM| = \sqrt{\left(\frac{10}{3}-1\right)^2 + \left(\frac{2}{3}-1\right)^2} = \sqrt{\frac{49}{9} + \frac{1}{9}} = \frac{5\sqrt{2}}{3}$

$m_{BC} = \frac{\frac{10}{3}}{-\frac{10}{3}} = -1 \qquad y = -x+4 \qquad |AH| = \frac{|1+1-4|}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$

$\frac{AM}{AH} = \frac{\frac{5\sqrt{2}}{3}}{\sqrt{2}} = \frac{5}{3}$

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$\frac{x}{A} + \frac{y}{B} = 1 \qquad -\frac{B}{A} = -\frac{1}{2} \qquad \frac{B}{A} = \frac{1}{2} \qquad B = \frac{A}{2}$

$\frac{x}{A} + \frac{2y}{A} = 1 \qquad x+2y-A = 0 \qquad$ فاصله از مبدا $= \frac{|0+0-A|}{\sqrt{5}} = \sqrt{20}$

$|A| = 10 \rightarrow |B| = 5 \qquad AB = \sqrt{A^2+B^2} = \sqrt{100+25} = 5\sqrt{5}$

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$y = \frac{4}{m}x + 2 \qquad$ طول از مبدا $= -2 \times \frac{m}{4} = -\frac{m}{2}$

$y = (1-m)x + 2 \qquad$ طول از مبدا $= \frac{-2}{1-m} = \frac{2}{m-1}$

قاعده $= \left|\frac{2}{m-1} + \frac{m}{2}\right| = \left|\frac{4+m^2-m}{2m-2}\right|$

در نقطه‌ای با طول ۲ یکدیگر را قطع کردند پس ارتفاع مثلث برابر با ۲ است

اندازه مساحت $= \frac{5}{2}$

$\frac{2 \times \text{قاعده}}{2} = \frac{5}{2} \rightarrow \left|\frac{m^2-m+4}{2m-2}\right| = \frac{5}{2}$

$\frac{m^2-m+4}{|m-1|} = 5$

$m^2-m+4 = 5m-5 \qquad m^2-6m+9 = 0 \qquad (m-3)^2 = 0 \qquad \boxed{m=3}$

$m^2-m+4 = -5m+5$

$m^2+4m-1 = 0 \qquad m = \frac{-4 \pm \sqrt{16+4}}{2} = -2 \pm \sqrt{5}$

$3 \times (-2+\sqrt{5})(-2-\sqrt{5}) = 3(4-5) = -3$

نقطه ی $\begin{vmatrix} 4 \\ 1 \end{vmatrix} A$ روی خط $y = 2x - 3$ قرار دارد

مساحت فوق اولیه و مساحت آن برابر است

مساحت A بعد از m $= \begin{vmatrix} 4 \\ -4 \end{vmatrix} - \begin{vmatrix} 2 \\ 1 \end{vmatrix} \cdot \begin{vmatrix} 2 \\ -6 \end{vmatrix}$

$2(x-2) = y + 6 \qquad y = 2x - 9$

$m = 1 \longrightarrow m' = -1 \qquad -(x-1) = y - 2 \qquad y = -x + 3$

$y = -x + 3$
$y = x - 6$ $\Big\} \rightarrow -2x = -9 \Rightarrow x = \frac{9}{2}$ $\qquad 2x - 2 = x \quad x = 1$

$y = 2$ $\qquad m \begin{vmatrix} 1 \\ 2 \end{vmatrix}$

$A' = 2m - A = \begin{vmatrix} -2 \\ 8 \end{vmatrix} - \begin{vmatrix} 1 \\ 2 \end{vmatrix} \cdot \begin{vmatrix} -2 \\ 6 \end{vmatrix}$ $\qquad -2 = 9 - 3$

$b = 6$ $\qquad 2b - 3 = 12 + 3 = 15$