بوینه ماشترکان

①

$A(1,4)$

الف) $A_2(8,-2)$

$m=\frac{\Delta y}{\Delta x}=\frac{y_2-y_1}{x_2-x_1}=\frac{4}{-1}=-4$

$y=ax+b \longrightarrow y=-4x+b \xrightarrow{(4,2)} 2=-4\times4+b$

$y=-4x+18$

$b=18$

ب) $2y+6x=-1$ خط موازی

$A(1,4)$ $m_2=-3$ $\longrightarrow y=-3x+b \xrightarrow{(4,2)}$

$m_1 \longrightarrow 2y+6x+1=0$ $\quad m_2=m_1=\frac{-\text{ضریب } x}{\text{ضریب } y}=\frac{-6}{2}=-3$ موازی

$2=-3\times4+b$

$2y=-6x-1$

$\Rightarrow y=-3x-\frac{1}{2}$ $\quad \boxed{y=-3x+14}$ $\quad \boxed{b=14}$

$=6+4$

ج) $x+3y=1$ عمود $\quad m_1\times m_2=-1 \longrightarrow -\frac{1}{3}\times m_2=-1$

$m_1 \longrightarrow \frac{-\text{ضریب } x}{\text{ضریب } y}=\frac{-1}{3}$ $\quad \boxed{m_2=3}$

$y=3x+b \xrightarrow{(4,2)}$

$3y=-x+1 \Rightarrow y=-\frac{1}{3}x+\frac{1}{3}$

$12+b=2$

$\boxed{y=3x-10}$ $\quad \boxed{b=-10}$

$y=ax+b$

د) $m=\tan\frac{\pi}{3}=\sqrt{3}$ $\quad y=\sqrt{3}x+b \xrightarrow{(4,2)} 2=4\sqrt{3}+b$

$b=2-4\sqrt{3}$

$y=\sqrt{3}x+2-4\sqrt{3}$

②

الف) فاصله دو نقطه $=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}=\sqrt{(2-(-1))^2+(3-7)^2}=\sqrt{25}=5$

ب) $3x+4y=3$ $\quad$ فاصله نقطه از خط $=\frac{|ax+by+c|}{\sqrt{a^2+b^2}}=\frac{|3\times2+4\times3-3|}{\sqrt{9+16}}=\frac{15}{5}=3$

(۳)

الف)

$$\begin{cases} 2x+3y=6 \longrightarrow 4x+6y=\boxed{12}\ c \\ 4x+6y=\boxed{8}\ c' \end{cases} \Longrightarrow \boxed{4x+6y=10}$$

دو خط موازی

$$\frac{2}{4}=\frac{3}{6}\neq\frac{6}{8}$$

منطبق نیستند

$$4x+6y=c$$

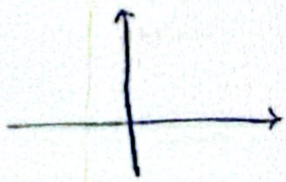
$$4x+6y=\frac{c+c'}{2}=\frac{12+8}{2}=10$$

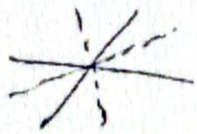
ب)

$$\frac{|c-c'|}{\sqrt{a^2+b^2}}=\frac{|12-8|}{\sqrt{4^2+6^2}}=\frac{4}{\sqrt{52}}=\frac{4}{2\sqrt{13}}=\frac{2}{\sqrt{13}}=\boxed{\frac{2\sqrt{13}}{13}}$$

$(16+36=52)$

(۴)





دو خط‌ها

$$\frac{|ax+by+c|}{\sqrt{a^2+b^2}}$$

$$3\times(2x+3y=3)\longrightarrow 6x+9y=9$$

$$2\times(3x-2y=1)\xrightarrow{\ominus} 6x-4y=2$$

$$13y=7 \Rightarrow \boxed{y=\frac{7}{13}}$$

$$6x-4\times\frac{7}{13}=2$$

$$6x=\frac{28}{13}+\frac{26}{13}=\frac{54}{13}=6x \Rightarrow \boxed{x=\frac{9}{13}} \quad x=\frac{9}{13}$$

$$\frac{|2x+3y-3|}{\sqrt{4+9}}=\frac{|3x-2y-1|}{\sqrt{9+4}}$$

$$|2x+3y-3|=|3x-2y-1|$$

① $\Rightarrow 2x+3y-3=3x-2y-1 \Rightarrow \boxed{5y=x+2} \Rightarrow \boxed{y=\frac{x}{5}+\frac{2}{5}}$

② $2x+3y-3=-3x+2y+1$

$$\boxed{y=-5x+4}$$

(۵)

$$y-2x=5 \xrightarrow{\text{شیب}} y=\boxed{2}x+5$$

$$y+3x=3 \xrightarrow{\text{شیب}} y=\boxed{-3}x+3$$

$$\tan(\alpha\pm\beta)=\frac{\overset{m_1}{\tan\alpha}\pm\overset{m_2}{\tan\beta}}{1\mp\tan\alpha\tan\beta} \Longrightarrow$$

⊕

$$\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}=\left|\frac{5}{1+(2\times-3)}\right|=\left|\frac{5}{-5}\right|=+1$$

زاویه بین دو خط

$$(\alpha-\beta)=45^\circ$$

⑥

الف) فاصله دو نقطه $= \sqrt{(x_1-x_2)^2+(y_1-y_2)^2} = \sqrt{\underbrace{(3-(-5))^2}_{64} + \underbrace{(-2-4)^2}_{36}} = 10$

ب) $x = \frac{x_A+x_B}{2} = \frac{3-5}{2} = \frac{-2}{2} = -1$

$y = \frac{y_A+y_B}{2} = \frac{4-2}{2} = 1$ $\quad (-1,1)$

⑦

الف) $G \Rightarrow \frac{x_1+x_2+x_3}{3} = x_G \qquad \frac{y_1+y_2+y_3}{3} = y_G$

$\frac{-9}{3} \quad \frac{3-2-10}{3} = -3 \qquad \frac{-9}{3} \quad \frac{1+2-12}{3} = -3 \qquad G=(-3,-3)$

ب) $\frac{1}{2}\begin{bmatrix} 3 & 1 & 1 \\ -2 & 4 & 1 \\ -10 & -12 & 1 \end{bmatrix}$ ① × ②

$\begin{vmatrix} \underbrace{9}_{3\times3\times1} + \underbrace{-10}_{1\times1\times-10} + \underbrace{24}_{-2\times-12\times1} \\ \underbrace{-(-30)}_{+30} \quad \underbrace{-(-9)}_{+9} \quad \underbrace{-0(-2)}_{2} \end{vmatrix} = 94$

$S_{\text{مثلث}} = \frac{1}{2}\times 94 = \boxed{47}$

⑧ الف) y برحسب x $\quad -y = \frac{2x+1}{4x-2} \Rightarrow y = \frac{2x+1}{3-4x}$

ب) x برحسب y $\quad y = \frac{-2x+1}{-4x-3}$

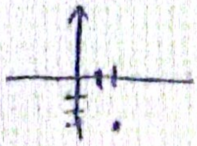
ج) x برحسب y $\quad x = \frac{2y+1}{4y-3} \Rightarrow$ طرفین وسطین $\Rightarrow y = -\frac{-3x-1}{4x-2} = \boxed{\frac{3x+1}{4x-2} = y}$

$\Rightarrow 4yx-3x = 2y+1 \Rightarrow 4yx-2y = 3x+1 \Rightarrow y(4x-2) = 3x+1 \Rightarrow \boxed{y = \frac{3x+1}{4x-2}}$

د) y برحسب x $\quad -x = \frac{-2y+1}{-4y-3} \Rightarrow$ طرفین وسطین $\Rightarrow y = -\frac{3x-1}{4x+2} = \boxed{\frac{-3x+1}{4x+2} = y}$

$\Rightarrow -4yx-3x = -2y+1 \Rightarrow -4yx+2y = 3x+1 \Rightarrow y(-4x+2) = 3x+1 \Rightarrow \boxed{y = \frac{3x+1}{-4x+2}}$

① $y = \dfrac{2x+1}{x-3}$



الف) $\begin{vmatrix} 2 \\ -3 \end{vmatrix}$ $\boxed{x' = x-2}$ $\cdot(x_0, y_0)$

$y' = y-(-3) = \boxed{y+3}$

$\Rightarrow \boxed{x = x'+2} \;,\; \boxed{y = y'-3}$

جایگذاری $\longrightarrow$

$y'-3 = \dfrac{2(x'+2)+1}{x'+2-3} = \dfrac{2x'+4+1}{x'-1} = \dfrac{2x'+5}{x'-1}$

$\Rightarrow y' = \dfrac{2x'+5}{x'-1} + 3 \longrightarrow \dfrac{3x'-3}{x'-1} + \dfrac{2x'+5}{x'-1} = \dfrac{5x'+2}{x'-1}$

$\boxed{y = \dfrac{5x+2}{x-1}}$

ب) $\begin{vmatrix} 3 \\ 2 \end{vmatrix}$ $x' = x-3 \Rightarrow x = x'+3$ جایگذاری $\longrightarrow$

$y' = y-2 \Rightarrow y = y'+2$

$y'+2 = \dfrac{2(x'+3)+1}{x'+3-3} = \dfrac{2x'+6+1}{x'} = \dfrac{2x'+7}{x'}$

$\Rightarrow y' = \dfrac{2x'+7}{x'} - 2 = \dfrac{2x'+7-2x'}{x'} = \dfrac{7}{x'}$

$\boxed{y = \dfrac{7}{x}}$

⑩

الف) حذفی بالایی − پایینی

$\begin{cases} 3x+4y=2 \\ (x-5y=1)\times 3 = 3x-15y=3 \end{cases} \ominus \Rightarrow 3x+4y-3x+15y = 2-3 = -1$

$\Rightarrow 19y = -1 \Rightarrow \boxed{y = \dfrac{-1}{19}}$

$x-5y=1 \longrightarrow x-5\times\dfrac{-1}{19} = 1 \Rightarrow x+\dfrac{5}{19} = 1$ $\boxed{x = \dfrac{14}{19}}$

ب) کرامر

$\begin{cases} 3x+4y=2 \\ x-5y=1 \end{cases}$ $x = -\dfrac{4-(-5\times 2)}{3\times(-5)-4\times 1} = \dfrac{+14}{-19} = \dfrac{14}{19}$

$y = \dfrac{3\times1-2\times1}{3\times(-5)-4\times1} = \boxed{\dfrac{1}{-19}}$

مخرج $\begin{bmatrix} 3 & 4 \\ 1 & -5 \end{bmatrix} = 3\times(-5)-4\times1 = -19$

صورت y = $\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} = 3\times1-2\times1 = 1$

صورت x = $\begin{bmatrix} 4 & 2 \\ -5 & 1 \end{bmatrix} = 4\times1-(-5)\times2 = 14$