

$y = -x^2 + bx + 12$ $y_{max} = \frac{-b}{2a} = \frac{-b^2 + 4ac}{4a} = \frac{-b^2 + 4(-1)(12)}{4(-1)} = \frac{-b^2 - 48}{-4} = \frac{b^2 + 48}{4} = 7 \rightarrow b^2 + 48 = 28$ $b^2 = 14 \rightarrow b = \pm \sqrt{14}$ $b = +\sqrt{14} \rightarrow y = -x^2 + \sqrt{14}x + 12 \xrightarrow{\Delta = b^2 - 4ac} \Delta = 14 - 4(-1)(12) > 0 \quad \text{دو جواب دارد}$ $b = -\sqrt{14} \rightarrow y = -x^2 - \sqrt{14}x + 12 \xrightarrow{\Delta = b^2 - 4ac} \Delta = 14 - 4(-1)(12) > 0 \quad \text{دو جواب دارد}$	۱
الف) $x^2 + 2x + 3 = 0 \xrightarrow{x_2 = t} t^2 + 2t + 3 = 0 \xrightarrow{\Delta = b^2 - 4ac} \Delta = 4 - 4(1)(3) < 0$ عبارت همواره مثبت و در هیچ نقطه ای صفر نمی شود ب) $f(x) = (5 - x^2)^2 - 2(5 - x^2) - 15 \xrightarrow{5 - x^2 = t} t^2 - 2t - 15 = 0 \rightarrow (t - 5)(t + 3) = 0$ ۵ $t = 5 \rightarrow 5 - x^2 = 5 \rightarrow x^2 = 0 \rightarrow x = 0$ عبارت همواره مثبت و در هیچ نقطه ای صفر نمی شود $t = -3 \rightarrow 5 - x^2 = -3 \rightarrow x^2 = 8 \rightarrow x = \pm \sqrt{8}$	۲
$5x^2 - 14x + m = 0$ $\alpha = \beta + 2 \rightarrow \alpha - \beta = 2$ $\alpha + \beta = \frac{-b}{a} = \frac{14}{5} = 2.8$ $\begin{cases} \alpha - \beta = 2 \\ \alpha + \beta = 2.8 \end{cases} \rightarrow \alpha = 4 \rightarrow \alpha = 3, \beta = 1$ $x = \alpha = 3 \xrightarrow{f(3)} \frac{5(3)^2 - 14(3) + m}{5} = 0 \rightarrow -14 + m = 0 \rightarrow m = 14$ ۵x² - 14x + 14 = 0 ✓	۳
$3\alpha^2 - 4\alpha + m - 1 = 0 \quad S = \alpha + \beta = \frac{-b}{a} \rightarrow S = \frac{4}{3} = 1.33$ $\begin{cases} 3\alpha - \beta = 5 \\ \alpha + \beta = 2 \end{cases} \xrightarrow{+} 4\alpha = 7 \rightarrow \alpha = 1.75$ $x = \alpha = 1.75 \xrightarrow{f(1.75)} 3(1.75)^2 - 4(1.75) + m - 1 = 0$ $\rightarrow m - 1 = 0 \rightarrow m = 1$ ۳x² - 4x = 0 ✓	۴
$-mx^2 + 5x + m^2 - 2 = 0 \quad p = \alpha\beta = 1 \rightarrow \frac{c}{a} = 1$ $\frac{c}{a} = \frac{m^2 - 2}{-m} = 1 \rightarrow m^2 - 2 = -m \rightarrow m^2 + m - 2 = 0 \rightarrow (m + 2)(m - 1) = 0$ مقادیر $m = -2$ و $m = 1$ در شرایط مسئله قرار می گیرند. $m = -2 \rightarrow 2x^2 + 5x + 2 = 0 \rightarrow x^2 + 2x + 1 = 0 \rightarrow (x + 1)^2 = 0 \rightarrow x = -1$ $m = 1 \rightarrow -x^2 + 5x - 1 = 0 \xrightarrow{\Delta = b^2 - 4ac} \Delta = 25 - 4(-1)(-1) > 0 \rightarrow \Delta > 0$ دو ریشه متمایز $m = 1$	۵

$$x^r - mx + r = 0 \rightarrow p = \frac{c}{a} \rightarrow \alpha \cdot \beta = r$$

$$\alpha \beta^r = r \rightarrow (\alpha \beta) \beta = r \rightarrow \beta = \frac{r}{\alpha}$$

1, VO

$$x = \beta \cdot \frac{r}{\alpha} \rightarrow \left(\frac{r}{\alpha}\right)^r - m \left(\frac{r}{\alpha}\right) + r = 0 \rightarrow \frac{14}{9} - \frac{r \cdot 14m}{r \cdot 9} + \frac{r}{9} = 0 \rightarrow \frac{14 - 14m + r}{9} = 0$$

$$\rightarrow -14m + r = 0 \rightarrow m = \frac{r}{14} \quad x^r - \frac{r}{14}x + r = 0 \quad \Delta > 0 \quad \checkmark$$

$$x^r - 5x + m = 0 \rightarrow S = \frac{-b}{a} \rightarrow \alpha + \beta = r \rightarrow 5\beta = r \rightarrow \beta = 1$$

$$\alpha = r\beta$$

$$x = \beta = 1 \rightarrow (1)^r - 5(1) + m = 0 \rightarrow 1 - 5 + m = 0 \rightarrow m = 4$$

$$x^r - 5x + 4 = 0 \quad \Delta > 0 \quad \checkmark$$

1, VO

$$x^r - vx + r = 0 \quad p = \frac{c}{a} \rightarrow \alpha \beta = r \rightarrow \beta = \frac{r}{\alpha} \rightarrow \beta^r = \frac{r}{\alpha^r}$$

$$\alpha^r + \frac{1}{\alpha^r} = ? \rightarrow \alpha^r + \frac{1}{\alpha^r} = \left(\alpha + \frac{r}{\alpha}\right) \left(\alpha^r - r + \frac{r}{\alpha^r}\right) \rightarrow (\alpha + \beta) (\alpha^r + \beta^r - r) = v (r + r) = 2r$$

$$S = \frac{-b}{a} \rightarrow \alpha + \beta = v$$

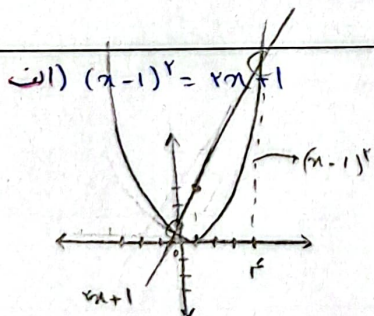
$$\alpha^r + \beta^r = S^r - rp = (v)^r - r \left(\frac{r}{v}\right) = 2r$$

$$h(t) = -1.5t^2 + 0.5t \quad h(t) = 0 \rightarrow -1.5t^2 + 0.5t = 0 \rightarrow -t^2 + 0.5t = 0 \rightarrow t(-t + 0.5) = 0$$

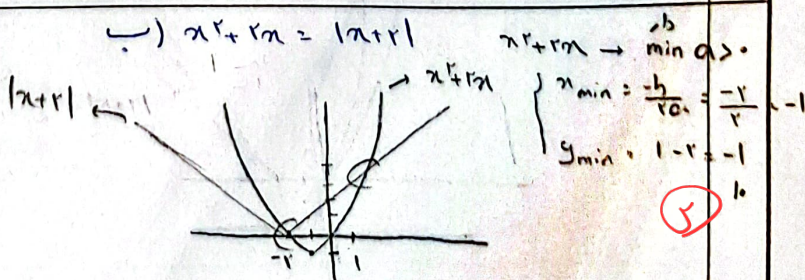
$$\begin{cases} \text{max } t: t_{\max} = \frac{-b}{2a} \rightarrow t_{\max} = \frac{0.5}{2(-1.5)} = \frac{0.5}{-3} = -\frac{1}{6} \\ \text{max } h: h_{\max} = \frac{(-b)^2 - 4ac}{4a} \rightarrow h_{\max} = \frac{(-0.5)^2 - 4(-1.5)(0)}{4(-1.5)} = \frac{0.25}{-6} = -\frac{1}{24} \end{cases}$$

$$\begin{cases} t = 0 \\ t = 0.5 \end{cases}$$

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$x=0, x=1$
نقطه‌های تقاطع منحنی با محورهای مختصات



$x=-1, x=1$
نقطه‌های تقاطع منحنی با محورهای مختصات

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