۱۹,۷۵

-۱
$y = -x^2 + bx + 3$

$-\frac{\Delta}{4a} = 7 \quad \frac{b^2+12}{4} = 7 \Rightarrow b^2 + 12 = 28 \Rightarrow b^2 = 16 \Rightarrow \boxed{b = \pm 4}$ (۲)

-۲
الف) $f(x) = x^4 + 2x^2 + 3$

$x^4 + 2x^2 + 3 = 0 \Rightarrow t^2 + 2t + 3 = 0 \quad t = \frac{-2 \pm \sqrt{4-12}}{2} \rightarrow \Delta < 0$ (این تابع صفر (ریشه) ندارد.) (۲)

$x^2 = t$

ب) $f(x) = (4-x^2)^2 - 2(4-x^2) - 15$

$(4-x^2)^2 - 2(4-x^2) - 15 = 0 \Rightarrow t^2 - 2t - 15 = 0 \quad (t-5)(t+3) = 0$

$t = 5 \Rightarrow 4 - x^2 = 5 \Rightarrow x^2 = -1$ غ‌ق‌ق

$t = -3 \Rightarrow 4 - x^2 = -3$

$x^2 = 7 \Rightarrow \boxed{x = \pm\sqrt{7}}$ ✓

$4 - x^2 = t$

-۳
$4x^2 - 16x + m = 0 \quad \alpha + \beta = \frac{16}{4} = 4$

$\alpha = \beta + 2 \quad \beta + 2 + \beta = 4 \Rightarrow 2\beta = 2 \Rightarrow \boxed{\beta = 1,\ \alpha = 3}$ (۱,۷۵)

$P = \frac{m}{4} \Rightarrow (3)(1) = \frac{m}{4} \Rightarrow \boxed{m = 12}$

$4x^2 - 16x + 12 = 0$
$\Delta > 0$ ✓

-۴
$3x^2 - 6x + m - 1 = 0$

$2\alpha - \beta = 4 \Rightarrow \frac{1}{2}\alpha + \frac{3}{2}\alpha + \frac{1}{2}\beta - \frac{3}{2}\beta = 4 \Rightarrow \frac{1}{\cancel{2}}(\overset{2}{\alpha+\beta}) + \frac{3}{2}(\alpha - \beta) = 4$ (۲)

$\frac{3}{2}(\alpha - \beta) = \cancel{3} \Rightarrow \alpha - \beta = 2$

$\alpha - \beta = \frac{\sqrt{\Delta}}{|a|} \Rightarrow \frac{\sqrt{36 - 12(m-1)}}{3} = 2 \quad \sqrt{36 - 12(m-1)} = 6 \Rightarrow \cancel{36} - 12(m-1) = \cancel{36}$

$m - 1 = 0 \Rightarrow \boxed{m = 1} \quad 3x^2 - 6x = 0 \quad 36 - 12 > 0$
$\Delta > 0$

-۵
$-mx^2 + 4x + m^2 - 2 = 0 \quad 16 + 4m^3 - 8m > 0 \quad 16 - 32 + 16 = 0$ (۲)

$\Delta > 0 \Rightarrow 16 + 4(m)\underbrace{(m^2-2)}_{m^3-2m} > 0 \quad 16 + 4 - 8 > 0$

$P = 1 \quad \frac{m^2-2}{-m} = 1 \Rightarrow m^2 - 2 = -m \Rightarrow m^2 + m - 2 = 0 \quad (m+2)(m-1) = 0$

$\boxed{m = 1}$ ✓ $\Delta > 0$

$m = -2$ غ‌ق‌ق $\Delta > 0$

-۶
$x^2 - mx + 3 = 0 \quad \alpha\beta^2 = 4 \Rightarrow \underbrace{(\alpha)(\beta)}_{3}(\beta) = 4 \Rightarrow \beta = \frac{4}{3}$

$m = \frac{9}{4} + \frac{4}{3} = \frac{27+16}{12} = \boxed{\frac{43}{12}}$ ✓ $\quad \alpha \times \frac{4}{3} = \cancel{3} \Rightarrow \alpha = \frac{9}{4}$ (۲)

$x^2 - \frac{43}{12}x + 3 = 0$

$\left(\frac{43}{12}\right)^2 - 12 > 0$

-۷

$x^2-4x+m=0$ $\quad \alpha+\beta=4$ $\quad x^2-4x+3=0$

$\alpha=3\beta$ $\quad 4\beta=4 \Rightarrow \beta=1$ $\quad 16-12\geq 0$

$m=\alpha\beta=3\times 1=\boxed{3}$ $\quad \alpha=3$ $\quad 5>0$

۲

-۸

$x^2-7x+2=0$

$$\alpha^3+\frac{8}{\alpha^3}=\underbrace{\left(\alpha+\frac{2}{\alpha}\right)}_{7}\underbrace{\left(\alpha^2+\frac{4}{\alpha^2}-2\right)}_{43}^{+6-6}=\boxed{301}$$

$\alpha+\frac{2}{\alpha}=\frac{\alpha^2+2}{\alpha}=\frac{7\cancel{\alpha}-\cancel{2}+\cancel{2}}{\cancel{\alpha}}=7$ $\quad \hookrightarrow \underbrace{\left(\alpha+\frac{2}{\alpha}\right)}_{7}^2-\cancel{6}=49-6=43$

$\alpha^2=7\alpha-2$

$\left(\alpha+\frac{2}{\alpha}\right)^2=\alpha^2+\frac{4}{\alpha^2}+4$

۲

-۹

$h(t)=-10t^2+50t$ $\quad -10t(t-5)=0$ $\quad t=0,5$

$\frac{-50}{-20}=\boxed{\frac{5}{2}}\rightarrow t$

$-\cancel{10}\left(\frac{25}{\cancel{4}_{2}}\right)+50\cdot\left(\frac{5}{2}\right)=\frac{-125+250}{2}=\frac{125}{2}$

۲

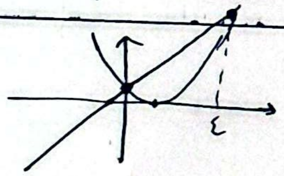
-۱۰

الف) $(x-1)^2=2x+1$

$y=2x+1$ $\quad$ $\frac{x}{y}$

$x^2-2x+\cancel{1}=2x+\cancel{1}$

$x^2-4x=0$ $\quad x(x-4)=0$



۲ تا جواب داره

$x=0$

$x=4$

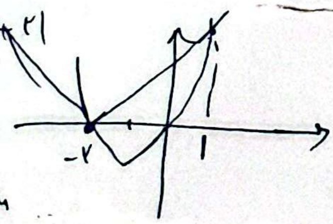
۲ جواب

$x=-2$

$x=1$

ب) $x^2+2x=|x+2|$

$x(x+2)$



$x+2=x^2+2x$

$x+2=-x^2-2x$

$x^2+3x+2=0$

$(x+2)(x+1)=0$ $\left\langle \begin{array}{l} x=-2 \checkmark \\ x=-1 \text{ غ ق ق} \end{array}\right.$

$x^2+x-2=0$

$(x+2)(x-1)=0$ $\left\langle \begin{array}{l} x=1 \checkmark \\ x=-2 \checkmark \end{array}\right.$

۲