

سوال (۱)

$$y = (a-1)x^2 + x + 3$$

$$\text{شرط تارن} \rightarrow \frac{-b}{2a} = 2 \rightarrow \frac{-1}{2(a-1)} = 2 \rightarrow -1 = 4(a-1) \rightarrow a-1 = -\frac{1}{4} \rightarrow a = -\frac{1}{4} + 1 \rightarrow a = \frac{3}{4}$$

$$\rightarrow y = (\frac{3}{4} - 1)x^2 + x + 3 \rightarrow y = -\frac{1}{4}x^2 + x + 3 = -(\frac{x}{4} + 1)(\frac{x}{4} - 3) \rightarrow \boxed{x = 4}$$

سوال (۲)

$$f(x) = mx^2 - 5x + m$$

$$\begin{array}{c|c|c} x & x_1 & x_2 \\ \hline f(x) & - & + \end{array}$$

$$\Rightarrow m < 0$$

$$\Delta > 0 \rightarrow (-5)^2 - 4(m)(m) > 0 \rightarrow 25 - 4m^2 > 0 \rightarrow$$

$$25 > 4m^2 \rightarrow \frac{25}{4} > m^2 \rightarrow -\frac{5}{2} < m < \frac{5}{2}$$

$$m < 0 \Rightarrow \boxed{m: (-\frac{5}{2}, 0)}$$

سوال (۳)

$$\text{ریشه ها: } \frac{3-\sqrt{5}}{3}, \frac{3+\sqrt{5}}{3}$$

$$a + \beta = \frac{3-\sqrt{5}}{3} + \frac{3+\sqrt{5}}{3} = \frac{3-\sqrt{5}+3+\sqrt{5}}{3} = \frac{6}{3} \rightarrow a + \beta = 2$$

$$a \cdot \beta = (\frac{3-\sqrt{5}}{3})(\frac{3+\sqrt{5}}{3}) = \frac{9-5}{9} = \frac{4}{9} \rightarrow a/\beta = \frac{4}{9}$$

$$\rightarrow x^2 - 2x + \frac{4}{9}$$

$$f(x) = 9x^2 - 18x + 4$$

سوال (۴)

$$2x^2 - 18x + m + 2 = 0 \quad a < \beta, \quad 2a^2 + \beta^2 = 12 \quad m = ? \quad S = \frac{-b}{a} = 9$$

$$a + \beta = \frac{18}{2} = 9, \quad a\beta = \frac{m+2}{2}, \quad 2a^2 + \beta^2 = 12 \rightarrow 2a^2 + a^2 + \beta^2 - \beta^2 = 2(a^2 + \beta^2) + (a^2 - \beta^2)$$

$$\rightarrow 2(S^2 - 2a\beta) + (S)(\frac{\sqrt{4}}{a}) = 2(14 - \frac{m+2}{1}) + \frac{2}{9}(\frac{\sqrt{4(1-m)}}{2}) =$$

$$\sqrt{(1-m)} + \frac{1}{9}\sqrt{4(1-m)} = 14 \rightarrow 14 - m + \frac{2}{9}\sqrt{14-m} = 14 \rightarrow \frac{2}{9}\sqrt{14-m} = m-14 \Rightarrow \sqrt{14-m} = \frac{9(m-14)}{2}$$

$$\rightarrow m^2 - 14m + 14 = 0 \rightarrow (m-7)^2 = 0 \rightarrow \boxed{m=7}$$

سوال (۵)

$$S(2, 9)$$

$$\text{ممکنه محور عرض را از نقطه ای به عرض ۵ قطع می کند} \Rightarrow C = 5$$

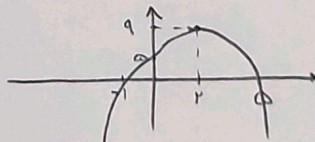
$$ax^2 + bx + C \xrightarrow{C=5} ax^2 + bx + 5 \xrightarrow{x=2, y=9} 9 = a(2)^2 + b(2) + 5 \rightarrow 4a + 2b = 4$$

$$\text{ممکنه از نقطه (۲, ۹)} \rightarrow \frac{-b}{2a} = 2 \rightarrow -b = 4a$$

$$4a + 2b = 4 \xrightarrow{4a = -b} -b + 2b = 4 \rightarrow b = 4, \quad 4a = -b \rightarrow 4a = -4 \rightarrow a = -1$$

$$\text{ممکنه} \rightarrow -x^2 + 4x + 5 = -(x-5)(x+1)$$

$$\begin{array}{c|c|c|c|c} x & 0 & 1 & 2 & -1 \\ \hline y & 5 & 4 & 9 & 0 \end{array}$$



بازه ای که ممکنه
برای این معادله
است

$$\boxed{(-1, 5)}$$

سوال (۶)

$$ax^2 - 7x + 4\beta = 0 \quad \beta > 0 \quad \frac{1}{a} + \frac{1}{\beta} = ?$$

$$a\beta = \frac{4\beta}{a} \rightarrow a^2 = 4 \rightarrow a = \pm 2$$

$$a + \beta = \frac{7}{a} \rightarrow \begin{cases} a=2 \rightarrow 2 + \beta = \frac{7}{2} \rightarrow \beta = \frac{7}{2} - 2 = \frac{3}{2} \quad \times \\ a=-2 \rightarrow -2 + \beta = -\frac{7}{2} \rightarrow \beta = -\frac{7}{2} + 2 = -\frac{3}{2} \quad \checkmark \end{cases} \rightarrow a = -2, \beta = \frac{3}{2}$$

$$\Rightarrow \frac{1}{a} + \frac{1}{\beta} = \frac{1}{-2} + \frac{1}{\frac{3}{2}} = -\frac{1}{2} + \frac{2}{3} = \frac{-3+4}{6} = \frac{1}{6}$$

سوال (۷)

$$x^2 + mx - 2m = 0 \quad a^2 - m\beta = 1 \quad a + \beta = ?$$

$$(a^2 - m\beta = 1) \rightarrow -ma + 2m - m\beta = 1 \rightarrow -m(a + \beta) = 1 - 2m \rightarrow m^2 = 1 - 2m \rightarrow m^2 + 2m - 1 = 0$$

$$-ma + 2m$$

$$-m$$

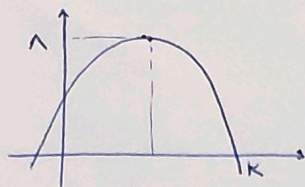
$$\rightarrow (m-2)(m+1) = 0 \rightarrow \begin{cases} m=-1 \rightarrow x^2 - 2x + 2 = 0 \\ m=2 \end{cases}$$

$$\Delta < 0$$

$$\text{ممکنه} \rightarrow x^2 + 2x - 4 \rightarrow a + \beta = S = -2$$

$$x^2 + 2x - 4 \rightarrow \Delta > 0 \quad \checkmark$$

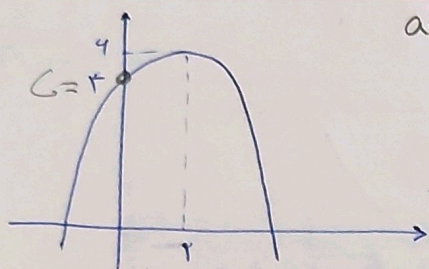
$$f(x) = mx^2 + kx + \frac{m}{4} + 1$$



سوال ۸

$$\begin{aligned} \frac{-\Delta}{4a} &= 1 \rightarrow -\frac{(14 - 4m)(\frac{m}{4} + 1)}{4m} = 1 \rightarrow -\frac{(14 - 2m^2 - 21m)}{4m} = 1 \rightarrow 2m^2 + 21m - 14 = 22m \\ &\rightarrow 2m^2 - 4m - 14 = 0 \rightarrow m^2 - 2m - 7 = 0 \rightarrow (m - 4)(m + 1) = 0 \rightarrow \begin{cases} m = 4 \rightarrow f(x) = 4x^2 + kx + 4 \rightarrow \Delta < 0 \\ m = -2 \rightarrow f(x) = -2x^2 + kx + 4 \rightarrow \Delta > 0 \end{cases} \\ &\Rightarrow f(x) = -2x^2 + kx + 4 \rightarrow x = 3, -1 \Rightarrow \boxed{k = 3} \end{aligned}$$

ریشه‌های معادله



$$ax^2 + bx + c$$

$$\frac{-b}{2a} = 2 \rightarrow b = -4a$$

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معادله تابع

$$\Rightarrow -1x^2 + 2x + 4$$

$$\frac{1}{a} + \frac{1}{b} = \frac{B+a}{aB} = \frac{S}{P} = \frac{-b}{\frac{c}{a}} = \frac{-b}{c} = \frac{-2}{4} = \frac{-1}{2} \Rightarrow \boxed{\frac{-1}{2}}$$

سوال ۹

$$x^2 - (4a+4)x + (3a^2+9a+3) = 0$$

$$\begin{aligned} &\rightarrow x^2 - 4(a+1)x + 3(a+1)^2 = 0 \rightarrow (x - (a+1))(x - 3(a+1)) = 0 \xrightarrow{\text{ریشه‌ها}} \begin{cases} a+1 \\ 3(a+1) \end{cases} \\ &\rightarrow \begin{cases} x - (a+1) = 0 \rightarrow x = a+1 \xrightarrow{x=2} 2 = a+1 \rightarrow a = 1 \xrightarrow{\text{ریشه‌ها}} \{2, 6\} \\ x - 3(a+1) = 0 \rightarrow x = 3(a+1) \xrightarrow{x=2} 2 = 3(a+1) \rightarrow a = -\frac{1}{3} \xrightarrow{\text{ریشه‌ها}} \{\frac{2}{3}, 2\} \end{cases} \end{aligned}$$

$$\text{ریشه‌های دیگر} = \boxed{4, 1, \frac{2}{3}}$$

سوال ۱۰