

تکلیف شماره ۱۳

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۱۹, ۷۵

$$y = (a-1)x^2 + x + 3, \quad x=2$$

$$x = \frac{-b}{2a} = \frac{-1}{2(a-1)} \rightarrow \frac{-1}{2(a-1)} = 2 \quad -1 = \epsilon a - \epsilon \rightarrow a = \frac{1}{\epsilon}$$

$$y = \frac{1}{\epsilon} x^2 + x + 3 \xrightarrow{x=2} x^2 - \epsilon x - 12 = 0 \rightarrow x = 9, -2 \rightarrow \boxed{x=9}$$

$$f(x) = mx^2 - \alpha x + m$$

$$\begin{array}{cc} x_1 & x_2 \\ - & + \\ \hline & \end{array}$$

$$\star \rightarrow \begin{cases} \Delta > 0 \rightarrow 4\alpha - 4m^2 > 0 \rightarrow m^2 < \frac{\alpha}{4} \\ m < 0 \end{cases} \rightarrow |m| < \frac{\sqrt{\alpha}}{2}, \frac{-\alpha}{4} < m < \frac{\alpha}{4}$$

$$\textcircled{1}, \textcircled{2} \xrightarrow{\text{استدلال}} \frac{\alpha}{4} < m < -\frac{\alpha}{4}$$

$$\text{نقطهٔ بحرانی: } \frac{1-\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}$$

$$S = \frac{1-\sqrt{5}}{2} + \frac{1+\sqrt{5}}{2} = 1$$

$$P = \frac{1+\sqrt{5}}{2} \times \frac{1-\sqrt{5}}{2} = -\frac{5}{4}$$

$$x^2 - Sx + P \rightarrow x^2 - 1x + \frac{5}{4} = 0 \xrightarrow{x=9} \boxed{9x^2 - 11x + 5 = 0}$$

$$1x^2 - 1x + m + 12 = 0, \quad \alpha < \beta, \quad 13\alpha^2 + \beta^2 \leq 17$$

$$S = \epsilon, \quad P = \frac{m+1}{2} \quad \alpha + \beta = \epsilon \rightarrow \beta = \epsilon - \alpha$$

$$13\alpha^2 + \beta^2 \leq 17 \Rightarrow 13\alpha^2 + (\epsilon - \alpha)^2 \leq 17 \Rightarrow 14\alpha^2 - 2\epsilon\alpha + \epsilon^2 \leq 17 \Rightarrow \epsilon^2 - 14\alpha + \epsilon \leq 0$$

$$\alpha^2 - 14\alpha + 1 \leq 0 \rightarrow (\alpha - 1)^2 \leq 0 \rightarrow \boxed{\alpha = 1} \quad \beta = \epsilon - \alpha \Rightarrow \epsilon - 1 \leq \beta \rightarrow \beta \leq 1$$

$$P \leq \alpha\beta \leq 1 \rightarrow \frac{m+1}{2} \leq 1 \rightarrow \boxed{m \leq 1}$$

$$S \mid \frac{1}{q} \quad (0, \omega) \quad y = a(x-h)^2 + k \rightarrow y = a(x-2)^2 + 9 \quad (\text{و.ج.})$$

$$0 = a(0-2)^2 + 9 \rightarrow a = -1 \quad y = -(x-2)^2 + 9 \rightarrow y) \rightarrow -(x-2)^2 + 9 = 0$$

$$(x-2)^2 < 9 \rightarrow -3 < x-2 < 3 \rightarrow \boxed{-1 < x < 5}$$

$$\alpha x^2 - 14x + 9\beta \leq 0, \quad \beta > 0, \quad \frac{1}{\alpha} + \frac{1}{\beta} \leq ?$$

$$S = \alpha + \beta \leq \frac{1}{\alpha}, \quad \alpha\beta \leq \frac{9}{\alpha} \rightarrow \alpha^2\beta \leq 9 \xrightarrow{\beta > 0} \alpha^2 \leq 9 \rightarrow \alpha = \pm 3$$

$$\alpha \leq 3 \rightarrow 3 + \beta \leq \frac{1}{3} \rightarrow \beta \leq -\frac{8}{3} \text{ غلط} \quad \alpha = -3 \rightarrow -3 + \beta \leq \frac{1}{-3} \rightarrow \boxed{\beta \leq -\frac{10}{3}}$$

$$\frac{1}{\alpha} + \frac{1}{\beta} \leq -\frac{1}{3} + \frac{3}{-10} = -\frac{13}{30} \leq \frac{1}{4} \quad \alpha = -3, \quad \beta \leq -\frac{10}{3}$$

$$x^2 + mx - 12 = 0, \quad \alpha^2 - m\beta \leq 1$$

$$\alpha^2 + m\alpha - 12 = 0$$

$$S = \alpha + \beta \leq -m$$

$$P = \alpha\beta \leq -12$$

$$\alpha^2 - m\beta \leq 1$$

$$(-m\alpha + 12) - m\beta \leq 1$$

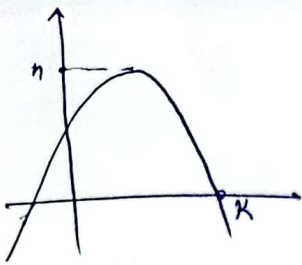
$$-m(\alpha + \beta) + 12 = 1 \leq 0$$

$$m \leq -\epsilon, \quad m \leq 1$$

$$m \leq -\epsilon \rightarrow 0 < \alpha$$

$$\alpha^2 - m\alpha + 12 = 0 \rightarrow \boxed{\alpha + \beta \leq -m}$$





$$f(x) = mx^2 + \epsilon x + \frac{m}{\epsilon} + V$$

$$x = \frac{-\epsilon}{2m} = \frac{-2}{m}$$

$$f(x) = m \left( \frac{-2}{m} \right)^2 + \epsilon \left( \frac{-2}{m} \right) + \frac{m}{\epsilon} + V = 1$$

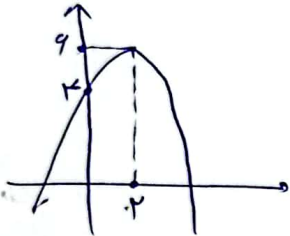
$$\frac{-\epsilon}{m} + \frac{m}{\epsilon} + V = 1 \implies -1 + m^2 = 2m$$

$$m^2 - 2m - 1 = 0 \implies m = 2 \pm \sqrt{5}$$

$$m = 2 \implies -2x^2 + \epsilon x + 4$$

$$x^2 + \epsilon x - 12 = 0 \implies (x+4)(x-3) = 0 \implies x = 3, x = -4$$

$$\checkmark \text{ جواب صحیح است } K = 3$$



$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{-b}{\frac{c}{a}} = \frac{-b}{c}$$

$$y = a(x-h)^2 + k \implies y = a(x-2)^2 + 4 \implies y = \epsilon = a(-2)^2 + 4$$

$$\implies a = \frac{1}{\epsilon} \implies y = \frac{1}{\epsilon} (x-2)^2 + 4 \implies \frac{1}{\epsilon} x^2 + \frac{4}{\epsilon} x + \epsilon = 0$$

$$\implies \frac{1}{\alpha} + \frac{1}{\beta} = \frac{-b}{c} = \frac{-\frac{4}{\epsilon}}{\epsilon} = \frac{-4}{\epsilon^2}$$

$$x^2 - (\epsilon a + \epsilon) x + (4a + 4 + \epsilon) = 0$$

$$\alpha = 2 \implies 5 = \alpha + \beta = \epsilon a + \epsilon \implies \beta = \epsilon a + 3$$

$$12 = \alpha\beta = 2a + 4a + 6 = 6a + 6 \implies 6a = 6 \implies a = 1$$

$$2a^2 - 2a - 1 = 0 \implies a^2 - 2a - \frac{1}{2} = 0 \implies a = \frac{2 \pm \sqrt{5}}{2}$$

$$a = 1 \implies x^2 - 1x + 12 = 0 \implies x = 3, x = 4$$

$$a = \frac{1}{\epsilon} \implies x^2 - \frac{1}{\epsilon} x + \frac{\epsilon}{\mu} = 0 \implies \mu x^2 - 1x + \epsilon = 0 \implies x = \frac{1}{\mu}, x = \epsilon$$

$$\boxed{x = 4 \text{ و } \frac{1}{\mu}}$$