نام و نام خانوادگی ......[illegible]...... **پاسخنامه تشریحی تکلیف شماره** ..۱۳.. **کلاس** ......یازدهم تجربی......

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$y = (a-1)x^2 + x + 3 \xrightarrow[x=2]{\text{متقارن نسبت به}} x_{ext} = \frac{-b}{2a} = \frac{-1}{2(a-1)} = 2 \rightarrow -1 = 4a - 4 \rightarrow 4a = 3 \rightarrow \boxed{a = \frac{3}{4}}$

$\Rightarrow (\underbrace{\tfrac{3}{4} - 1}_{-\frac{1}{4}})x^2 + x + 3 = 0 \xrightarrow{\times(-4)} x^2 - 4x - 12 = 0 \rightarrow (x-6)(x+2) = 0$

$\begin{cases} x = 6 \longrightarrow \text{ق ق ، طول(+)} \\ x = -2 \ \text{غ ق ق} \longrightarrow \text{طول(+) می خواهیم} \end{cases}$

این منحنی محور x ها را در $\boxed{x=6}$ قطع می کند

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$f(x) = mx^2 - 5x + m$

x		x_1		x_2	
$f(x)$	$-$	0	$+$	0	$-$

$\mathrm{I} \cap \mathrm{II}: \ -\frac{5}{2} < m < 0$

$\begin{cases} 1)\ a < 0 \longrightarrow m < 0 : \mathrm{I} \\ 2)\ \Delta > 0 \longrightarrow b^2 - 4ac > 0 \rightarrow 25 - 4(m)(m) > 0 \longrightarrow 25 - 4m^2 > 0 \rightarrow \frac{25}{4} > m^2 \rightarrow \frac{5}{2} > m > -\frac{5}{2} : \mathrm{II} \end{cases}$

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$x^2 - Sx + P = 0$

$S = \frac{3-\sqrt{5}}{3} + \frac{3+\sqrt{5}}{3} = \frac{\overbrace{3+3}^{6} - \cancel{\sqrt{5}} + \cancel{\sqrt{5}}}{3} = 2$

$P = \frac{3-\sqrt{5}}{3} \times \frac{3+\sqrt{5}}{3} = \frac{9-5}{3} = \frac{4}{3}$

$\Rightarrow x^2 - 2x + \frac{4}{3} = 0 \xrightarrow{\times 3} 3x^2 - 6x + 4 = 0$

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$2x^2 - 8x + (m+2) = 0 \rightarrow \alpha + \beta = \frac{-b}{a} = 4, \ \alpha\beta = \frac{c}{a} = \frac{m+2}{2}, \ \alpha^2 + \beta^2 = S^2 - 2P = 16 - m - 2 = 14 - m$

$3\alpha^2 + \beta^2 = 12 \Rightarrow 2\alpha^2 + \alpha^2 + 2\beta^2 - \beta^2 = 12 \Rightarrow 2(\alpha^2+\beta^2) + (\alpha-\beta)(\overbrace{\alpha+\beta}^{4}) = 12 \rightarrow \cancel{2}(14-m) + \cancel{4}^{2}(-\sqrt{12-2m}) = \cancel{12}^{6}$

$|\alpha - \beta| = \frac{\sqrt{\Delta}}{|a|} \rightarrow |\alpha - \beta| = \frac{\sqrt{64 - 4(2)(m+2)}}{2} \rightarrow \alpha - \beta = \frac{\cancel{2}\sqrt{12-2m}}{\cancel{2}} \rightarrow |\alpha - \beta| = \sqrt{12-2m} \xrightarrow{\alpha<\beta} \alpha - \beta = -\sqrt{12-2m}$

$\longrightarrow 14 - m - 2\sqrt{12-2m} = 6 \longrightarrow 8 - m = 2\sqrt{12-2m} \xrightarrow{(\ )^2} 64 + m^2 - 16m = \overbrace{4(12-2m)}^{48-8m} \longrightarrow m^2 - 8m + 16 = 0$

$\longrightarrow (m-4)^2 = 0 \longrightarrow m = 4$

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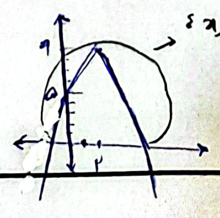
مختصات رأس سهمی $S(2,9)$: $\begin{cases} 1)\ x_{ext} = \frac{-b}{2a} = 2 \rightarrow \boxed{-b = 4a} \\ \text{عرض از مبدأ} \Rightarrow c = 5 \end{cases}$ $\quad 2)\ y_{ext} = \frac{-\Delta}{4a} = 9 \rightarrow \frac{-b^2 + \overbrace{4ac}^{-5b}}{\underbrace{4a}_{-b}} = 9 \rightarrow \frac{b^2 + 5b}{b} = 9$

$\rightarrow b^2 + 5b = 9b \rightarrow b^2 - 4b = 0 \rightarrow b(b-4) = 0 \rightarrow \begin{cases} b = 0 \ \text{غ ق ق} \\ b = 4 \ \text{ق ق} \rightarrow \boxed{a = -1} \longrightarrow -x^2 + 4x + 5 = 0 \end{cases}$

$\xrightarrow[-1+5=4]{\text{جمع ضرایب توان های زوج = فرد}} \begin{cases} x = -1 \\ x = \frac{-c}{a} = 5 \end{cases}$

در کدام بازه سهمی در بالای محور x ها $\Rightarrow D_f : (-1, 5)$



بالای محور x ها

$0<\beta$ و α و β ریشه‌های معادله

$\alpha x^2-7x+9\beta=0$

$S=\frac{-b}{a}\Rightarrow \alpha+\beta=\frac{7}{\alpha}\to \alpha^2+\alpha\beta=7\to \alpha\beta=-2$

$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{-1}{3}+\frac{3}{2}=\frac{-2+9}{6}=\frac{7}{6}$

$P=\frac{c}{a}\Rightarrow \alpha\cdot\beta=\frac{9\beta}{\alpha}\to \alpha^2=9\to$
- $\alpha=+3$ غ‌ق‌ق $\to -2=\frac{9\beta}{3}\to \beta=\frac{-2}{3}$ غ‌ق‌ق $\to 0<\beta$
- $\alpha=-3$ ق‌ق $\to -2=\frac{9\beta}{-3}\to 2=3\beta\to \beta=\frac{2}{3}$ ق‌ق

$k=\frac{x}{\text{منفی}}$

$f(x)=mx^2+4x+\left(\frac{m+16}{2}\right)$

$y_{max}=\frac{-\Delta}{4a}\to\frac{-b^2+4ac}{4a}=8\to\frac{-16+4(m)\left(\frac{m+16}{2}\right)}{2\cdot 4m}=8\to\frac{-8+m^2+16m}{2m}=8$ ۸

$\to m^2+16m-8=16m\to m^2-2m-8=0\to(m-4)(m+2)=0\to$
- $m=4\to$ غ‌ق‌ق $\times\ a<0$
- $m=-2\to$ ق‌ق $\checkmark$

نتیجه $y_{max}\to a<0\xrightarrow{m=-2}f(x)=-2x^2+4x+6\xrightarrow{f(x)=0}-2x^2+4x+6=0\to -x^2+2x+3=0$

جمع ضرایب توان زوج = فرد: $-1+3=2$
- $x=-1$
- $x=\frac{-c}{a}=3$

$\to x_{\text{منفی}}=k\to -1=k$

$x^2+mx-2m=0$

$\overset{(\alpha+\beta)}{S}\Rightarrow\frac{-b}{a}=-m,\quad \overset{(\alpha\cdot\beta)}{P}\Rightarrow\frac{c}{a}=-2m$

$S^2-2P=m^2-2(-2m)=m^2+4m$

$m^2+4m=S^2-2P$

$\alpha^2-m\beta=8\to\alpha^2+(\alpha+\beta)(\beta)=8\to\alpha^2+\beta^2+\overset{-2m}{\alpha\beta}=8\to m^2+2m=8\to m^2+2m-8=0$ ۷

$\to(m+4)(m-2)=0\to$ { $m=-4$ غ‌ق‌ق , $m=2$ ق‌ق } $\to x^2+2x-4=0\to\Delta=4-4(1)(-4)>0$

$\downarrow(\Delta=b^2-4ac)$

$x^2-4x+8=0$ غ‌ق‌ق

$\Delta=\underbrace{16-4(1)(8)}_{-32}<0$

$\to m=2\to\alpha+\beta=S\Rightarrow -m=-2$

$\alpha+\beta=-2$

عرض از مبدأ: $c=4$

$ax^2+bx+c=0\to -\frac{1}{2}x^2+2x+4=0$

$\Delta=b^2-4ac\to\Delta=4-4\left(-\frac{1}{2}\times 4\right)=4+8=12$

$x=\frac{-b\pm\sqrt{\Delta}}{2a}\to x=\frac{-2\pm2\sqrt{3}}{-1}=$
- $x_1=2-2\sqrt{3}$
- $x_2=2+2\sqrt{3}$

اکسترمم: ۹
- $x_{max}:\ \frac{-b}{2a}=2\to -b=4a$
- $y_{max}:\ \frac{-\Delta}{4a}=6\to\frac{+b^2+4ac}{4a}=6\to\frac{+b^2+4b}{+b}=6\to b^2+4b=6b\to b^2-2b=0\to b(b-2)=0\to$ { $b=0$ غ‌ق‌ق , $b=2$ ق‌ق }

$a=-\frac{1}{2}$

مجموع معکوس‌های ریشه‌ها:

$\frac{1}{x_1}+\frac{1}{x_2}=\frac{1}{2(1-\sqrt{3})}+\frac{1}{2(1+\sqrt{3})}=\frac{1+\sqrt{3}+1-\sqrt{3}}{(2)(1-3)}=\frac{2}{2\times-2}=\frac{-1}{2}$

$x^2-(\underbrace{4a+4}_{4(a+1)})x+(\underbrace{3a^2+6a+3}_{3(a+1)^2})=0\xrightarrow{a+1=t}x^2-(4t)x+(3t^2)=0\xrightarrow{x=2}4-8t+3t^2=0\to t^2-8t+12=0$

$(t-6)(t-2)=0$

- $t=\frac{6}{3}=2,\ (a+1=2\to a=1)\to x^2-8x+12=0\to(x-6)(x-2)=0\Rightarrow$ { $x=2$, $x=6$ } ۱۰
- $t=\frac{2}{3}\to\left(a+1=\frac{2}{3}\to a=-\frac{1}{3}\right)\to x^2-\frac{8}{3}x+\frac{4}{3}=0\to\left(x-\frac{2}{3}\right)(x-2)=0\Rightarrow$ { $x=2$, $x=\frac{2}{3}$ }