

$y = (\alpha - 1)x^2 + x + 3$ $\xrightarrow{x=2}$ $x_{ent} = \frac{-b}{2a} = \frac{-1}{2(\alpha-1)} = 2 \rightarrow -1 = 2\alpha - 4 \rightarrow 2\alpha = 3 \rightarrow \alpha = \frac{3}{2}$

$\Rightarrow (\frac{3}{2} - 1)x^2 + x + 3 = 0 \xrightarrow{x(-4)}$ $x^2 - 4x - 12 = 0 \rightarrow (x-4)(x+2) = 0$

$\begin{cases} x=4 \rightarrow \text{نقطه } (4, 3) \\ x=-2 \rightarrow \text{نقطه } (-2, 3) \end{cases}$

این منحنی دایره را در $x=4$ قطع می‌کند

$f(x) = mx^2 - 2x + m$

$I \cap II: \frac{-a}{r} < m < 0$

$\begin{cases} 1) a < 0 \rightarrow m < 0 : I \\ 2) a > 0 \rightarrow b^2 - 4ac > 0 \rightarrow 4 - 4(m)(m) > 0 \rightarrow 4 - 4m^2 > 0 \rightarrow \frac{4}{4} > m^2 \rightarrow \frac{2}{2} > m > -\frac{2}{2} : II \end{cases}$

$x^2 - 5x + p = 0$

$S = \frac{5 - \sqrt{5}}{2} + \frac{5 + \sqrt{5}}{2} = 5$

$P = \frac{5 - \sqrt{5}}{2} \times \frac{5 + \sqrt{5}}{2} = \frac{25 - 5}{4} = \frac{20}{4} = 5$

$\Rightarrow x^2 - 5x + 5 = 0 \rightarrow x^2 - 4x + 4 = 0$

$2m^2 - 11m + 14 = 0 \rightarrow \alpha + \beta = \frac{-b}{a} = \frac{11}{2}, \alpha\beta = \frac{c}{a} = \frac{14}{2} = 7, \alpha^2 + \beta^2 = 5^2 - 2p = 25 - 14 = 11$

$2\alpha^2 + 2\beta^2 = 22 \Rightarrow 2\alpha^2 + \alpha^2 + 2\beta^2 + \beta^2 = 22 \Rightarrow 3(\alpha^2 + \beta^2) + (\alpha - \beta)^2 = 22 \Rightarrow 3(11) + (\alpha - \beta)^2 = 22 \Rightarrow (\alpha - \beta)^2 = -11$

$|\alpha - \beta| = \sqrt{11} \rightarrow |\alpha - \beta| = \frac{\sqrt{44 - 4(2)(7)}}{2} \rightarrow \alpha - \beta = \frac{\sqrt{44 - 56}}{2} = \frac{\sqrt{-12}}{2} = \frac{i\sqrt{12}}{2} = i\sqrt{3}$

$\rightarrow \alpha - m - i\sqrt{3} = 0 \rightarrow \alpha - m = i\sqrt{3} \xrightarrow{1) \times} 4\alpha + m^2 - 14m = 11(1 - 3) \rightarrow m^2 - 11m + 14 = 0$

$\rightarrow (m-4)^2 = 0 \rightarrow m = 4$

$S(2, 9): \begin{cases} 1) x_{ent} = \frac{-b}{2a} = -2 \rightarrow -b = 4a \\ 2) y_{ent} = \frac{-d}{2a} = 9 \rightarrow \frac{-b^2 + 4ac}{4a} = 9 \rightarrow \frac{b^2 + 4b}{4} = 9 \rightarrow b^2 + 4b = 36 \rightarrow b^2 + 4b - 36 = 0 \end{cases}$

$\rightarrow b^2 + 4b - 36 = 0 \rightarrow b^2 - 4b = 0 \rightarrow b(b-4) = 0 \rightarrow \begin{cases} b=0 \\ b=4 \end{cases}$

$\rightarrow \begin{cases} b=0 \rightarrow \alpha = -1 \\ b=4 \rightarrow \alpha = -1 \end{cases} \rightarrow -x^2 + 4x + 5 = 0$

$\Rightarrow D_f: (-1, 5)$

$\beta > \alpha > 0 < \beta$ ملاحظه

$$\alpha x^2 - \gamma x + 9\beta = 0$$

$$\begin{cases} S = \frac{-b}{a} \Rightarrow \alpha + \beta = \frac{\gamma}{\alpha} \rightarrow \alpha^2 + \alpha\beta = \gamma \rightarrow \alpha\beta = -\gamma & \frac{1}{\alpha} + \frac{1}{\beta} = \frac{-1}{\gamma} + \frac{\gamma}{\gamma} = \frac{-\gamma + \gamma}{\gamma} = \frac{0}{\gamma} \\ P = \frac{c}{a} \Rightarrow \alpha \cdot \beta = \frac{9\beta}{\alpha} \rightarrow \alpha^2 = 9 \rightarrow \begin{cases} \alpha = +3 \rightarrow -\gamma = \frac{9\beta}{3} \rightarrow \beta = \frac{-\gamma}{3} \rightarrow \beta < 0 \\ \alpha = -3 \rightarrow -\gamma = \frac{9\beta}{-3} \rightarrow \gamma = 3\beta \rightarrow \beta = \frac{\gamma}{3} \rightarrow \beta > 0 \end{cases} \end{cases}$$

$$f(x) = mx^2 + \epsilon x + \left(\frac{m+\epsilon}{\gamma}\right)$$

$k = \frac{\gamma}{\epsilon}$

$$y_{max} = \frac{-D}{4a} \rightarrow \frac{-b^2 + 4ac}{4a} = \Lambda \rightarrow \frac{-\gamma^2 + 4\epsilon(m)(\frac{m+\epsilon}{\gamma})}{4\epsilon m} = \Lambda \rightarrow \frac{-\gamma^2 + 4\epsilon m(m+\epsilon)}{4\epsilon m} = \Lambda$$

$$\rightarrow m^2 + 4\epsilon m - \Lambda = 4\epsilon m \rightarrow m^2 - \gamma m - \Lambda = 0 \rightarrow (m - \gamma)(m + \gamma) = 0 \rightarrow \begin{cases} m = \gamma \rightarrow \text{Good} \\ m = -\gamma \rightarrow \text{Bad} \end{cases}$$

$$\begin{aligned} \text{Good} \rightarrow \alpha < 0 & \xrightarrow{m = -\gamma} f(x) = -\gamma x^2 + \epsilon x + \gamma \xrightarrow{f(x)=0} -\gamma x^2 + \epsilon x + \gamma = 0 \rightarrow -x^2 + \gamma x + \epsilon = 0 \\ \text{Bad} \rightarrow \alpha < 0 & \xrightarrow{m = -\gamma} f(x) = -\gamma x^2 + \epsilon x + \gamma \xrightarrow{f(x)=0} -\gamma x^2 + \epsilon x + \gamma = 0 \rightarrow -x^2 + \gamma x + \epsilon = 0 \end{aligned}$$

$$S \Rightarrow \frac{-b}{a} = -m, P \Rightarrow \frac{c}{a} = -\gamma m \quad \begin{aligned} \Delta = b^2 - 4ac &= \gamma^2 - 4\epsilon(-\gamma m) = \gamma^2 + 4\epsilon\gamma m \\ m^2 + 4\epsilon m &= \gamma^2 - \gamma^2 = 0 \end{aligned}$$

$$\alpha^2 - m\beta = \Lambda \rightarrow \alpha^2 + (\alpha + \beta)(\beta) = \Lambda \rightarrow \alpha^2 + \beta\alpha + \alpha\beta = \Lambda \rightarrow m^2 + \gamma m = \Lambda \rightarrow m^2 + \gamma m - \Lambda = 0$$

$$\rightarrow (m + \gamma)(m - \gamma) = 0 \rightarrow \begin{cases} m = -\gamma \rightarrow \text{Good} \\ m = \gamma \rightarrow \text{Bad} \end{cases} \rightarrow \alpha^2 + \gamma\alpha - \epsilon = 0 \rightarrow \Delta = \gamma^2 - 4(-\epsilon)(-\epsilon) > 0$$

$$\alpha^2 - \epsilon\alpha + \Lambda = 0 \rightarrow \Delta = 14 - 4(-\epsilon)(\Lambda) < 0$$

$$\alpha x^2 + bx + c = 0 \rightarrow \frac{-1}{\gamma} x^2 + \gamma x + \epsilon = 0 \rightarrow \Delta = b^2 - 4ac = \gamma^2 - 4(-\frac{1}{\gamma})(\epsilon) = \gamma^2 + \frac{4\epsilon}{\gamma} = 12$$

$$x = \frac{-b \pm \sqrt{\Delta}}{2a} \rightarrow x = \frac{-\gamma \pm \sqrt{12}}{-1} = \begin{cases} x_1 = \gamma - \sqrt{12} \\ x_2 = \gamma + \sqrt{12} \end{cases}$$

$$\frac{1}{x_1} + \frac{1}{x_2} = \frac{1}{\gamma(1-\sqrt{12})} + \frac{1}{\gamma(1+\sqrt{12})} = \frac{1+\sqrt{12}+1-\sqrt{12}}{\gamma(1-12)} = \frac{2}{\gamma(-11)} = -\frac{2}{11\gamma}$$

$$\begin{aligned} x^2 - (\gamma\alpha + \epsilon)x + (\gamma\alpha\gamma + 4\alpha + \gamma) &= 0 \xrightarrow{a+1=t} x^2 - (t)x + (t\gamma) = 0 \xrightarrow{x=\gamma} \gamma^2 - \Lambda t + \gamma t = 0 \rightarrow t^2 - \Lambda t + \gamma = 0 \\ t = \frac{\gamma}{\gamma} = 1 &\rightarrow (a+1=\gamma \rightarrow a=1) \rightarrow x^2 - \Lambda x + \gamma = 0 \rightarrow (x-4)(x-\gamma) = 0 \rightarrow \begin{cases} x=4 \\ x=\gamma \end{cases} \\ t = \frac{\gamma}{\gamma} = 1 &\rightarrow (a+\frac{\gamma}{\gamma} = \frac{\gamma}{\gamma} \rightarrow a=1) \rightarrow x^2 - \frac{\Lambda}{\gamma}x + \frac{\epsilon}{\gamma} = 0 \rightarrow (x-\frac{\gamma}{\gamma})(x-\frac{\epsilon}{\gamma}) = 0 \rightarrow \begin{cases} x=\gamma \\ x=\frac{\epsilon}{\gamma} \end{cases} \end{aligned}$$