باران کاظمی   یازدهم C   تکلیف شماره‌ی ۱۳

۱-

$y=(a-1)x^2+x+3 \quad (a-1=-\frac{1}{4})$

$\frac{-1}{2a-2}=2 \quad 4a-4=-1 \Rightarrow 4a=3 \Rightarrow a=\frac{3}{4}$

$\frac{-1\pm\sqrt{1+3}}{-\frac{1}{2}}$: $\frac{-1+2}{-\frac{1}{2}}=-2$ ✗ ، $\frac{-1-2}{-\frac{1}{2}}=\boxed{6}$

۲-

$f(x)=mx^2-5x+m$

$x_1,\ x_2$: $-\ |\ +\ |\ -$

$m<0$ ★

$\Delta>0 \rightarrow 25-4m^2>0 \quad 4m^2<25$

$m^2<\frac{25}{4} \rightarrow -\frac{5}{2}<m<\frac{5}{2}$ ★

$\xrightarrow{★\cap} -\frac{5}{2}<m<0 \quad (-\frac{5}{2},0)$

۳-

$\frac{3-\sqrt5}{3}\ ,\ \frac{3+\sqrt5}{3}$

$S=\frac{3-\sqrt5+3+\sqrt5}{3}=2$

$y=x^2-Sx+P$

$P=\frac{(3-\sqrt5)(3+\sqrt5)}{9}=\frac{9-5}{9}=\frac{4}{9}$

$y=x^2-2x+\frac{4}{9}$

۴-

$2x^2-8x+m+2=0 \qquad \alpha^2+\beta^2=S^2-2P=16-m-2=14-m$

$\alpha<\beta \qquad S=4 \quad P=\frac{m+2}{2} \qquad \alpha-\beta=\frac{\sqrt\Delta}{|a|}=\frac{\sqrt{64-8m-16}}{2}=\frac{\sqrt{48-8m}}{2}$

$3\alpha^2+\beta^2=12 \Rightarrow 2\alpha^2+\alpha^2+2\beta^2-\beta^2=12 \Rightarrow 2(\alpha^2+\beta^2)+(\alpha+\beta)(\alpha-\beta)=12$

$28-2m+2\sqrt{48-8m}=12$

$2m-16=2\sqrt{48-8m} \Rightarrow 4m^2+256=4(48-8m) \Rightarrow 4m^2+32m+64=0$

$m^2+8m+16=0 \quad (m+4)^2=0 \Rightarrow \boxed{m=-4}$

۵-

$S\,|\,2,\ 9$

$y=a(x-h)^2+k \Rightarrow y=a(x-2)^2+9 \rightarrow 5=a(4)+9 \Rightarrow 4a=-4 \Rightarrow a=-1$

$(x-2)^2=x^2-4x+4$

$y=-x^2+4x-4+9 \Rightarrow y=-x^2+4x+5$

$-(x^2-4x-5)=-(x-5)(x+1)$ ، $-1,\ 5$: $-\ |\ +\ |\ -$ ، $(-1,5)$

۶-

$\alpha x^2-7x+9\beta=0$

$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}=\frac{\frac{7}{\alpha}}{-2}=\boxed{\frac{7}{6}}$

$\alpha\beta^2-7\beta+9\beta=0$

$\alpha\beta^2=-2\beta \Rightarrow \alpha\beta=-2$

$\alpha\beta=\frac{9\beta}{\alpha} \Rightarrow \alpha^2=9$: $\alpha=3$ ✗ ، $\alpha=-3$

$\Rightarrow \beta=\frac{2}{3}$

۷-

$x^2 + mx - 2m = 0$

$\alpha^2 - m\beta = 8 \Rightarrow 2m - m\alpha - m\beta = 8$

$S = ?$ $\qquad 2m - m(\alpha+\beta) = 8 \Rightarrow 2m + m^2 = 8 \Rightarrow m^2 + 2m - 8 = 0$

$(m+4)(m-2) = 0$

$\xrightarrow{\alpha} \alpha^2 + m\alpha - 2m = 0 \Rightarrow \alpha^2 = 2m - m\alpha$

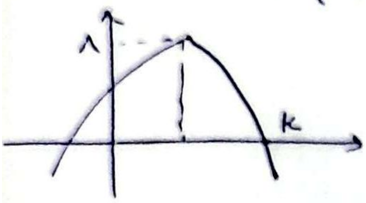
$m = 2$ ✓ $\quad x^2 + 2x - 4 = 0, \ \Delta = 4 + 16 > 0$ ✓

$m = -4$ غق $\quad x^2 - 4x + 8 = 0, \ 16 - 32 < 0$

$S = -m = \boxed{-2}$

۸-

$f(x) = mx^2 + 4x + \frac{m}{2} + 7$



$\frac{-\Delta}{4a} = 8 \Rightarrow \frac{-(16 - (4)(m)(\frac{m}{2}+7))}{4m} \Rightarrow \frac{-(16 - 2m^2 - 28m)}{4m} = 8$ $\qquad (2m^2 + 28m)$

$2m^2 + 28m - 16 = 32m$

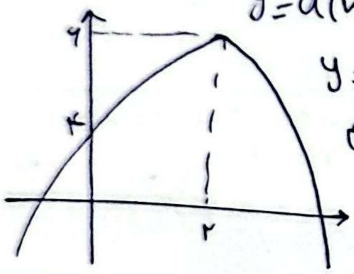
$2m^2 - 4m - 16 = 0 \Rightarrow m^2 - 2m - 8 = 0 \quad (m-4)(m+2) = 0$ → $m = 4$ غق، $\boxed{m = -2}$

$-2x^2 + 4x + 6 = 0$

$-x^2 + 2x + 3 = 0 \quad x^2 - 2x - 3 = 0 \quad (x-3)(x+1) = 0$ → $\boxed{x = 3 = k}$، $x = -1$

۹-

$y = a(x-h)^2 + k$ ~~$y = ax^2 + bx + c$~~



$y = a(x-2)^2 + 6 \Rightarrow y = -(x-2)^2 + 12$ $\qquad (x^2 - 4x + 4)$

$4 = a(4) + 6$

$4a = -2 \Rightarrow a = -\frac{1}{2}$

$y = -x^2 + 4x + 8 \Rightarrow y \quad -x^2 + 4x + 8 = 0 \Rightarrow x^2 - 4x - 8 = 0$

$\frac{4 \pm \sqrt{16+32}}{2} = \frac{4 \pm 4\sqrt{3}}{2} = 2 \pm 2\sqrt{3}$ → $\frac{1}{2+2\sqrt{3}}$، $\frac{1}{2-2\sqrt{3}}$

$\frac{1}{2+2\sqrt{3}} + \frac{1}{2-2\sqrt{3}} = \frac{2 - 2\sqrt{3} + 2 + 2\sqrt{3}}{-10} = -\frac{4}{10}$

$= \boxed{-\frac{2}{5}}$

۱۰-

$x^2 - (4a+4)x + (3a^2 + 6a + 3) = 0$

$4 - 2(4a+4) + 3a^2 + 6a + 3 = 0$

$4 - 8a - 8 + 3a^2 + 6a + 3 = 0 \Rightarrow 3a^2 - 2a - 1 = 0 \qquad \frac{2 \pm \sqrt{4+12}}{6} =$ 1، $-\frac{1}{3}$

$a = 1 \Rightarrow x^2 - 8x + 12 = 0$ ✓ $\Rightarrow (x-6)(x-2) = 0 \Rightarrow \boxed{x = 6}$

$a = -\frac{1}{3} \Rightarrow x^2 - (\frac{8}{3})x + \frac{4}{3} = 0$ ✓ $\Rightarrow \Delta = \frac{64}{9} - \frac{16}{3} = \frac{64-48}{9} = \frac{16}{9} \Rightarrow \frac{\frac{8}{3} \pm \frac{4}{3}}{2} = \frac{\frac{4}{3}}{\frac{2}{1}} = \boxed{\frac{2}{3}}$