-۱ (۲)

$y=(a-1)x^2+x+3$, $a-1=-\frac{1}{4}$

$\frac{-1}{2a-2}=2$ $\quad 4a-4=-1 \Rightarrow 4a=3 \Rightarrow a=\frac{3}{4}$

$\frac{-1\pm\sqrt{1+3}}{-\frac{1}{2}}$: $\frac{-1+2}{-\frac{1}{2}}=-2$ ✗ , $\frac{-1-2}{-\frac{1}{2}}=\boxed{6}$

-۲ (۲)

$f(x)=mx^2-5x+m$

x_1		x_2	
$-$	$+$	$-$	

$m<0$ ★

$\Delta>0 \rightarrow 25-4m^2>0$ $\quad 4m^2<25$

$m^2<\frac{25}{4} \rightarrow -\frac{5}{2}<m<\frac{5}{2}$ ★

$\xrightarrow{★\cap} -\frac{5}{2}<m<0 \quad (-\frac{5}{2},0)$

-۳ (۱٫۵)

$\frac{3-\sqrt{5}}{3}$, $\frac{3+\sqrt{5}}{3}$ $\qquad S=\frac{3-\sqrt{5}+3+\sqrt{5}}{3}=2$

$y=x^2-Sx+P$ $\qquad P=\frac{(3-\sqrt{5})(3+\sqrt{5})}{9}=\frac{9-5}{9}=\frac{4}{9}$

$y=x^2-2x+\frac{4}{9} \xrightarrow{\times 9}$ ضرایب صحیح

-۴ (۱)

$2x^2-8x+m+2=0$ $\qquad \alpha^2+\beta^2=S^2-2P=16-m-2=14-m$

$\alpha<\beta$ $\quad S=4 \quad P=\frac{m+2}{2}$ $\quad \alpha-\beta=\frac{\sqrt{\Delta}}{|a|}=\frac{\sqrt{64-8m-16}}{2}=\frac{\sqrt{48-8m}}{2}$

$3\alpha^2+\beta^2=12 \Rightarrow 2\alpha^2+\alpha^2+2\beta^2-\beta^2=12 \Rightarrow 2(\alpha^2+\beta^2)+(\alpha+\beta)(\alpha-\beta)=12$

$28-2m \; (+) \; 2\sqrt{48-8m}=12$ $\qquad 192-32m$

$2m-16=2\sqrt{48-8m} \Rightarrow 4m^2+256=4(48-8m) \Rightarrow 4m^2+32m+64=0$

$m^2+8m+16=0 \quad (m+4)^2=0 \Rightarrow m=+4$

-۵ (۲)

$S \mid \begin{matrix}2\\9\end{matrix}$

$y=a(x-h)^2+k \Rightarrow y=a(x-2)^2+9 \rightarrow 5=a(4)+9 \Rightarrow 4a=-4 \Rightarrow a=-1$

$(x-2)^2 = x^2-4x+4$

$y=-x^2+4x-4+9 \Rightarrow y=-x^2+4x+5$

$-(x^2-4x-5)=-(x-5)(x+1)$

	-1		5	
$-$		$+$		$-$

$(-1,5)$

-۶ (۲)

$\alpha x^2-7x+9\beta=0$

$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}=\frac{-\frac{7}{\alpha}}{-2}=\boxed{\frac{7}{6}}$

$\alpha\beta^2-7\beta+9\beta=0$, 2β

$\alpha\beta^2=-2\beta \Rightarrow \alpha\beta=-2$

$\alpha\beta=\frac{9\beta}{\alpha} \Rightarrow \alpha^2=9$ ، $\alpha=3$ ✗ ، $\alpha=-3$

$\Rightarrow \beta=\frac{2}{3}$

-۷

$x^2+mx-2m=0$

$\alpha^2-m\beta=1 \Rightarrow 2m-m\alpha-m\beta=0$

$S=?$ $\quad 2m-m(\alpha+\beta)=0 \Rightarrow 2m+m^2=0 \Rightarrow m^2+2m-1=0$ (۲)

$(m+3)(m-2)=0$

$m=2$ ✓ $\quad 2x^2+4x+1=0$ ✓

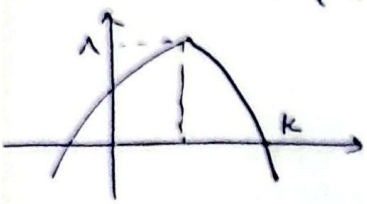
$m=-3$ غ ق $\quad 9x^2-6x+1=0$

$\alpha \rightarrow \alpha^2+m\alpha-2m=0 \Rightarrow \alpha^2=2m-m\alpha$

$S=-m=\boxed{-2}$

-۸

$f(x)=mx^2+3x+\frac{m}{2}+1$



$\frac{-\Delta}{4a}=8 \Rightarrow \frac{-(9-4(m)(\frac{m}{2}+1))}{4m} \Rightarrow \frac{-(9-2m^2-4m)}{4m}=8$ (۲)

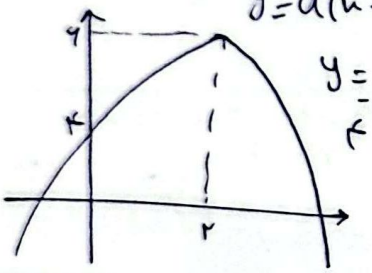
$2m^2+4m-9=32m$

$2m^2-28m-9=0 \Rightarrow m^2-14m-4=0 \quad (m-7)(m+2)=0$ $\begin{cases} m=7 \text{ غ ق} \\ m=-2 \end{cases}$

$-x^2+4x+5>0$ $\quad x^2-4x-5<0$ $\quad (x-5)(x+1)<0$ $\begin{cases} x<5 \\ x>-1 \end{cases}$ $\Rightarrow \boxed{x<5}$

-۹

$y=a(x-h)^2+k$



$y=a(x-2)^2+4 \Rightarrow y=(x-2)^2+1$ (۱/۵)

$4=a(4)+4$

$4a=-2 \Rightarrow a=-\frac{1}{2}$

$y=-x^2+4x+8 \Rightarrow y=-x^2+4x+8=0 \Rightarrow x^2-4x-8=0$

$\frac{4\pm\sqrt{16+32}}{2}=\frac{4\pm4\sqrt{2}}{2}=2\pm2\sqrt{2}$ $\begin{cases} \frac{1}{2+2\sqrt{2}} \\ \frac{1}{2-2\sqrt{2}} \end{cases}$

$\frac{1}{2+2\sqrt{2}}+\frac{1}{2-2\sqrt{2}}=\frac{2-2\sqrt{2}+2+2\sqrt{2}}{-4}=\frac{-4}{4}$

$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}=\frac{4}{-8}=\frac{-1}{2}$

$=\boxed{-\frac{2}{a}}$

-۱۰

$x^2-(4a+4)x+(3a^2+4a+3)=0$

$4-2(4a+4)+3a^2+4a+3=0$

$4-8a-8+3a^2+4a+3=0 \Rightarrow 3a^2-4a-1=0$ (۲)

$\frac{4\pm\sqrt{16+12}}{6}=\begin{cases} 1 \\ -\frac{1}{3} \end{cases}$

$a=1 \Rightarrow x^2-8x+12=0$ ✓ $\Rightarrow (x-6)(x-2)=0 \Rightarrow \boxed{x=6}$

$a=-\frac{1}{3} \Rightarrow x^2-(\frac{8}{3})x+\frac{4}{9}=0$ ✓ $\Rightarrow \Delta=\frac{64}{9}-\frac{16}{9}=\frac{64-16}{9}=\frac{48}{9}$

$\frac{8}{3}\pm\frac{\frac{8}{3}}{2}=\frac{\frac{16}{3}}{\frac{2}{1}}=\boxed{\frac{8}{3}}$