بنام خدا

کمار ریاضی

یازدهم دفتر C

تکلیف شماره ۱۳

۱

$y=(a-1)x^2+x+3=0$

$x=2 \to x_s=\frac{-b}{2a}=2 \to 4a-4=-1 \to 4a=3 \to a=\frac{3}{4}$

$x_s=\frac{-1}{2a-2}=2$

$y=-\frac{1}{4}x^2+x+3=0$

$x=\frac{-1\pm\sqrt{1+4\times\frac{1}{4}\times 3}}{2\times -\frac{1}{4}}$ → $\frac{-1+2}{-\frac{1}{2}}=-2$, $\frac{-3}{-\frac{1}{2}}=\boxed{+6}$

۲

$f(x)=mx^2-5x+m$

x		x_1		x_2	
$f(x)$	$-$	0	$+$	0	$-$

$m<0$ I

$\Delta>0 \to 25-4m^2>0$

$m^2<\frac{25}{4} \to \begin{cases} m<\frac{5}{2} & \text{II} \\ m>-\frac{5}{2} & \text{III} \end{cases}$

I , II , III $\Rightarrow \boxed{-\frac{5}{2}<m<0}$

۳

$y=x^2-Sx+P$

$S=\frac{3-\sqrt{5}}{3}+\frac{3+\sqrt{5}}{3}=2$

$P=\left(\frac{3-\sqrt{5}}{3}\right)\left(\frac{3+\sqrt{5}}{3}\right)=\frac{9-5}{9}=\frac{4}{9}$

$y=x^2-2x+\frac{4}{9} \xrightarrow{\times 9} \boxed{9y=9x^2-18x+4}$

۴

$y=2x^2-8x+m+2=0$ $\quad \alpha<\beta \quad 3\alpha^2+\beta^2=12$

$S=\frac{8}{2}=4 \to \beta+\alpha=4 \to \beta=4-\alpha$ $\quad 3\alpha^2+(4-\alpha)^2=12 \to 4\alpha^2-8\alpha+4=0$

$\alpha^2-2\alpha+1=0$

$(\alpha-1)^2=0 \to \alpha=1$

$\to \beta=4-1=3$

$P=\frac{c}{a}=\alpha.\beta \Rightarrow \frac{m+2}{2}=3\times 1 \Rightarrow \boxed{m=6-2=4}$

Scanned with CamScanner

(۵)

$C=5$

$S \,|\, \begin{matrix} 2 \\ 9 \end{matrix}$

$y=ax^2+bx+c$

$$\left.\begin{matrix} \frac{-b}{2a}=2 \to b=-4a \\ C=5 \end{matrix}\right\} \to ax^2-4ax+5 \xrightarrow{(2,9)} 4a-8a+5=9$$

$a=-1$

$y=-x^2+4x+5$ ↳ ریشه: ۵ و ۱-

	-1		5	
$-$		$+$		$-$

↳ $\boxed{(-1,5)}$

در این بازه بالای محور x ها است

(6)

$\alpha x^2-7x+9\beta=0 \qquad \beta>0$

$\alpha^3-7\alpha+9\beta=0$

$\alpha\beta^2-7\beta+9\beta=0 \to \alpha\beta^2+2\beta=0$

$\beta(\alpha\beta+2)=0 \xrightarrow{\beta\neq 0} \alpha\beta=-2$

$P=\frac{C}{a}=\frac{9\beta}{\alpha}=-2 \to 9\beta=-2\alpha \to \alpha^3-7\alpha+9\beta=0 \xrightarrow{9\beta=-2\alpha} \alpha^3-9\alpha=0$

$\alpha(\alpha^2-9)=0$

$\hookrightarrow \alpha \to 0,\ -3,\ 3$

$9\beta=-2\alpha \xrightarrow{\beta>0,\ \beta\neq 0} \alpha<0 \to \alpha=-3$

$\hookrightarrow \alpha\beta=-2 \xrightarrow{\alpha=-3} \beta=\frac{2}{3}$

$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\beta+\alpha}{\alpha\beta}=\frac{2/3-3}{-2}=\frac{-7/3}{-2}=\boxed{\frac{7}{6}}$

(7)

$x^2+mx-2m=0 \qquad \alpha^2-m\beta=8 \qquad S=-m$
$P=-2m$

$$\ominus\begin{cases}\alpha^2+m\alpha-2m=0 \\ \beta^2+m\beta-2m=0\end{cases}$$

$\alpha^2-\beta^2+m\alpha-m\beta=0$

$$8-\beta^2+m\alpha=0 \to \oplus\begin{cases}\beta^2-m\alpha=8 \\ \alpha^2-m\beta=8\end{cases}$$

$\beta^2+\alpha^2-m(\alpha+\beta)=16$

$\hookrightarrow S^2-2P-m(S)=16$

$m^2+4m+m^2-16=0$

$2m^2+4m-16=0$

$m^2+2m-8=0 \to m \to -4,\ +2$

$\Delta<0$

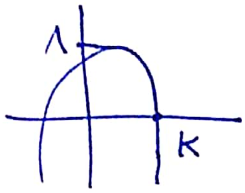
غ $\quad m=-4 \to x^2-4x+8=0$

ق $\quad m=2 \to x^2+2x-4=0$

$\boxed{S=-2}$

8

$f(x) = mx^2 + 4x + \frac{m}{2} + 7$



$y_s = 8$

$x_s = \frac{-4}{2m} = \frac{-2}{m} \rightarrow y_s = m \times \frac{4}{m^2} + \frac{m}{2} + 7 = 0$

$\frac{8 - 16 + m^2 + 14m}{2m} = 8$

$m^2 + 14m - 8 = 16m$

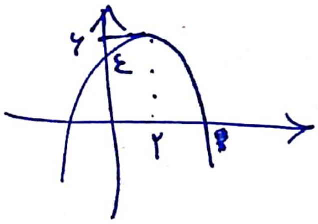
$m^2 - 2m - 8 = 0 \rightarrow m \begin{cases} -2 \\ 4 \end{cases}$

$x_s > 0 \rightarrow \frac{-2}{m} > 0 \rightarrow m < 0 \rightarrow m = -2$

$y = -2x^2 + 4x - 1 + 7 \Rightarrow f(x) = -2x^2 + 4x + 6 = 0 \rightarrow \begin{cases} -1 \\ 3 \end{cases}$

$\boxed{K = 3} \leftarrow K > 0$

9



$x_s = 2 \rightarrow -b/2a = 2 \rightarrow b = -4a$

$y_s = 6$

$c = 4$

$y = ax^2 + bx + c$

$y = ax^2 - 4ax + 4 \xrightarrow{(2,6)}$

$4a - 8a + 4 = 6 \rightarrow -4a = 2 \rightarrow a = -1/2$

$y = -\frac{1}{2}x^2 + 2x + 4$

$\hookrightarrow \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{4}{-8} = \boxed{-\frac{1}{2}}$

10

$x^2 - (4a+4)x + (3a^2 + 6a + 3) = 0$

$x = 2 \rightarrow 4 - 2(4a+4) + 3a^2 + 6a + 3 = 0 \rightarrow 3a^2 - 2a - 1 = 0 \rightarrow a \begin{cases} 1 \\ -1/3 \end{cases}$

$a = 1 \rightarrow x^2 - 8x + 12 \longleftrightarrow x \begin{cases} 2 \\ \boxed{6} \end{cases} \quad (x-6)(x-2) = x^2 - 8x + 12$

$a = -1/3 \rightarrow x^2 - 8/3\, x + 4/3 = 0 \rightarrow \Delta = \frac{64}{9} - \frac{16}{3} = \frac{16}{9} \rightarrow x = \frac{\frac{8}{3} \pm \frac{4}{3}}{2} = \begin{cases} 6/3 = 2 \\ \boxed{2/3} \end{cases}$