تیمار ریاضی — بنام خدا — ۲۰

یازدهم دفتر C

تکلیف شماره ۱۳

۱)

$y=(a-1)x^2+x-3=0$

$x=2 \rightarrow x_s=\frac{-b}{2a}=2 \rightarrow 4a-4=-1 \rightarrow 4a=3 \rightarrow a=\frac{3}{4}$

$x_s=\frac{-1}{2a-2}=2$

$y=-\frac{1}{4}x^2+x-3=0$

$x_{1,2}=\frac{-1\pm\sqrt{1+4\times\frac{1}{4}\times 3}}{2\times -\frac{1}{4}} \nearrow \frac{-1+2}{-1/2}=-2 \quad \searrow \frac{-3}{-1/2}=+6$

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۲)

$f(x)=mx^2-5x+m$

x		x_1		x_2	
$f(x)$	$-$	0	$+$	0	$-$

$m<0$ (I)

$\Delta>0 \rightarrow 25-4m^2>0$

$m^2<\frac{25}{4} \rightarrow \begin{cases} m<\frac{5}{2} & \text{(II)} \\ m>-\frac{5}{2} & \text{(III)} \end{cases}$

(I), (II), (III) $\Rightarrow -\frac{5}{2}<m<0$

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۳)

$y=x^2-Sx+P \qquad S=\frac{3-\sqrt{5}}{3}+\frac{3+\sqrt{5}}{3}=2$

$P=\left(\frac{3-\sqrt{5}}{3}\right)\left(\frac{3+\sqrt{5}}{3}\right)=\frac{9-5}{9}=\frac{4}{9}$

$y=x^2-2x+\frac{4}{9} \xrightarrow{\times 9} 9y=9x^2-18x+4$

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۴)

$y=2x^2-8x+m+2=0 \qquad \alpha<\beta \qquad 3\alpha^2+\beta^2=12$

$S=\frac{8}{2}=4 \rightarrow \beta+\alpha=4 \rightarrow \beta=4-\alpha \qquad 3\alpha^2+(4-\alpha)^2=12 \rightarrow 4\alpha^2-8\alpha+4=0$

$\alpha^2-2\alpha+1=0$

$(\alpha-1)^2=0 \Rightarrow \alpha=1$

$P=\frac{c}{a}=\alpha.\beta \Rightarrow \frac{m+2}{2}=3\times 1 \Rightarrow m=6-2=4 \qquad \hookrightarrow \beta=4-1=3$

۵) ۲)

$C=5$

$S \,|\, \begin{matrix} 2 \\ 9 \end{matrix}$

$y=ax^2+bx+c$

$\left.\begin{matrix} \frac{-b}{2a}=2 \to b=-4a \\ c=5 \end{matrix}\right\} \to ax^2-4ax+5 \xrightarrow{(2,9)} 4a-8a+5=9$

$a=-1$

$y=-x^2+4x+5 \to$ ریشه: ۵ و ۱-

-1		5	
−	+		−

$\to (-1,5)$

در این بازه بالای محور x ها است

۶) ۲)

$\alpha x^2 - 7x + 9\beta = 0 \qquad \beta > 0$

$\alpha^3 - 7\alpha + 9\beta = 0$

$\alpha\beta^2 - 7\beta + 9\beta = 0 \to \alpha\beta^2 + 2\beta = 0$

$\beta(\alpha\beta+2)=0 \xrightarrow{\beta\neq 0} \alpha\beta=-2$

$P=\frac{c}{a}=\frac{9\beta}{\alpha}=-2 \to 9\beta=-2\alpha \to \alpha^3-7\alpha+9\beta=0 \xrightarrow{9\beta=-2\alpha} \alpha^3-9\alpha=0$

$\alpha(\alpha^2-9)=0$

$\alpha \to 0,\ -3,\ 3$

$9\beta=-2\alpha \xrightarrow{\beta>0,\ \beta\neq 0} \alpha<0 \to \alpha=-3$

$\to \alpha\beta=-2 \xrightarrow{\alpha=-3} \beta=\frac{2}{3}$

$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\beta+\alpha}{\alpha\beta}=\frac{2/3-3}{-2}=\frac{-7/3}{-2}=\frac{7}{6}$

۷) ۲)

$x^2+mx-2m=0 \qquad \alpha^2-m\beta=8 \qquad S=-m$

$P=-2m$

$\ominus\begin{cases} \alpha^2+m\alpha-2m=0 \\ \beta^2+m\beta-2m=0 \end{cases}$

$\alpha^2-\beta^2+m\alpha-m\beta=0$

$8-\beta^2+m\alpha=0 \to \oplus\begin{cases} \beta^2-m\alpha=8 \\ \alpha^2-m\beta=8 \end{cases}$

$\beta^2+\alpha^2-m(\alpha+\beta)=16$

$\to S^2-2P-m(S)=16$

$m^2+4m+m^2-16=0$

$2m^2+4m-16=0$

$m^2+2m-8=0 \to m \to -4,\ +2$

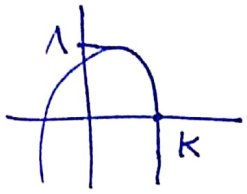
$\Delta<0$ ✓ $m=-4 \to x^2-4x+8=0$

✓ $m=2 \to x^2+2x-4=0$

$S=-2$

Scanned with CamScanner

$f(x) = mx^2 + 4x + \frac{m}{2} + V$



$y_s = 1$

$x_s = \frac{-4}{2m} = \frac{-2}{m} \rightarrow y_s = m \times \frac{4}{m^2} + \frac{m}{2} + V = 0$

$\frac{8 - 16 + m^2 + 8m}{2m} = 1$

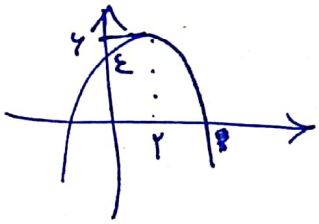
$m^2 + 16m - 8 = 12m$

$m^2 - 2m - 8 = 0 \rightarrow m \begin{cases} -2 \\ 4 \end{cases}$

$x_s > 0 \rightarrow \frac{-2}{m} > 0 \rightarrow m < 0 \rightarrow m = -2$

$y = -2x^2 + 4x - 1 + V \Rightarrow f(x) = -2x^2 + 4x + 2 = 0 \rightarrow \begin{cases} -1 \\ 3 \end{cases}$

$K = 3 \leftarrow K > 0$



$x_s = 2 \rightarrow -b/2a = 2 \rightarrow b = -4a$

$y_s = 4$

$C = 4$

$y = ax^2 + bx + c$

$y = ax^2 - 4ax + 4 \quad (2,4)$

$4a - 8a + 4 = 4 \rightarrow -4a = 0 \rightarrow a = -1/2$

$y = -\frac{1}{2}x^2 + 2x + 4$

$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{4}{-8} = -1/2$

$x^2 - (4a+4)x + (3a^2 + 2a + 3) = 0$

$x = 2 \rightarrow 4 - 2(4a+4) + 3a^2 + 2a + 3 = 0 \rightarrow 3a^2 - 6a - 1 = 0 \rightarrow a \begin{cases} 1 \\ -1/3 \end{cases}$

$a = 1 \rightarrow x^2 - 8x + 12 \longleftrightarrow x = \begin{cases} 2 \\ 6 \end{cases} \quad (x-6)(x-2) = x^2 - 8x + 12$

$a = -1/3 \rightarrow x^2 - \frac{1}{3}x + \frac{5}{3} = 0 \rightarrow \Delta = \frac{64}{9} - \frac{12}{3} = \frac{16}{9} \rightarrow x = \frac{\frac{1}{3} \pm \frac{4}{3}}{2} = \begin{cases} \frac{5}{3} > 2 \\ \frac{-3}{3} \end{cases}$