نام و نام خانوادگی : آیلین عظیمی   تحصیلی : ۱۲   کلاس : تجربه یازدهم   (۲۰)

۱ (۲)

$y = (a-1)x^2 + x + 3$

طول راس $= 2 \rightarrow -\frac{b}{2a} = 2 \rightarrow \frac{-1}{2a-2} = 2 \rightarrow 4a-4 = -1$

ریشه ها = ?

$4a = 3 \rightarrow a = \frac{3}{4}$

$y = -\frac{1}{4}x^2 + x + 3 \xrightarrow{\times 4} x^2 - 4x - 12$

$(x-6)(x+2) \rightarrow \frac{6}{-2} = (6)$

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۲ (۲)

$f(x) = mx^2 - 5x + m$

x		x_1		x_2	
$f(x)$	$-$		$+$		$-$

$\rightarrow \begin{cases} \Delta > 0 \rightarrow 25 - 4m^2 > 0 \rightarrow 4m^2 < 25 \quad (I) \\ m < 0 \quad (II) \end{cases}$

$m^2 < \frac{25}{4} \rightarrow -\frac{5}{2} < m < \frac{5}{2}$

$(I) \cap (II) \rightarrow \boxed{m \in (-\frac{5}{2}, 0)}$

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۳ (۲)

ریشه ها : $\frac{3+\sqrt{5}}{3}, \frac{3-\sqrt{5}}{3}$

ضرب ریشه ها $= \frac{9-5}{9} = \frac{4}{9} = P$

جمع ریشه ها : $\frac{3+\sqrt{5}+3-\sqrt{5}}{3} = 2 = S$

$\Rightarrow y = x^2 - Sx + P : y = x^2 - 2x + \frac{4}{9} \xrightarrow{\times 9} \boxed{y = 9x^2 - 18x + 4}$

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۴ (۲)

$2x^2 - 8x + m + 2 = 0 \quad \alpha, \beta \rightarrow \alpha < \beta \rightarrow S = \frac{-b}{a} = \frac{8}{2} = 4 \quad P = \frac{c}{a} = \frac{m+2}{2}$

$\alpha - \beta = \frac{-\sqrt{\Delta}}{a} = \frac{\sqrt{64 - 8m - 16}}{2} = -\frac{\sqrt{48-8m}}{2}$

$3\alpha^2 + \beta^2 = 12 \rightarrow 2\alpha^2 + \alpha^2 + 2\beta^2 - \beta^2 = 2(\alpha^2 + \beta^2) + (\alpha-\beta)(\alpha+\beta)$

$S^2 - 2P = 16 - m - 2 = 14 - m$

$\Rightarrow 28 - 2m + 4\left(-\frac{\sqrt{48-8m}}{2}\right) = 28 - 2m - 2\sqrt{48-8m} = 12$

$2\sqrt{48-8m} = -2m + 16$

$\sqrt{48-8m} = -m + 8$ به توان ۲

$48 - 8m = m^2 + 64 - 16m$

$m^2 - 8m + 16 = 0 \rightarrow (m-4)^2 = 0 \rightarrow \boxed{m = 4}$

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۵ (۲)

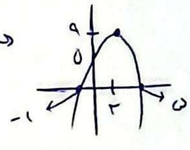
راس $\begin{bmatrix} 2 \\ 9 \end{bmatrix} \Rightarrow y = ax^2 + bx + 5$

$c = 5 \rightarrow \frac{-b}{2a} = 2 \rightarrow -b = 4a \rightarrow 4a + b = 0$

$\rightarrow f(2) = 9 \rightarrow 4a + 2b + 5 = 9 \rightarrow 4a + 2b = 4$

$b = 4, \quad a = -1$

$y = -x^2 + 4x + 5$



ریشه ها : -1 , $\frac{-c}{a} = \frac{-5}{-1} = 5$

$\boxed{\text{ریشه ها } (-1, 5)}$

$\alpha x^2 - 7x + 9\beta = 0 \qquad \alpha, \beta$

$\to S: -\frac{b}{a} = \frac{7}{\alpha} = \alpha+\beta \to \beta = \frac{7}{\alpha} - \alpha = \frac{7-\alpha^2}{\alpha} = \beta$

$P: \frac{9\beta}{\alpha} = \alpha\beta \to \alpha^2 = 9, \ \alpha = 3 \to \beta = \frac{7-9}{3} = -\frac{2}{3} \ \alpha$

$\alpha = -3 \to \beta = \frac{7-9}{-3} = \frac{-2}{-3}, \ \frac{2}{3} = \beta$

$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{\frac{7}{\alpha}}{\frac{9\beta}{\alpha}} = \frac{7}{9\beta} = \frac{7}{9(\frac{2}{3})} = \boxed{\frac{7}{6}}$ ② 6

$x^2 + mx - 2m = 0 \leadsto \alpha^2 = 2m - m\alpha \qquad S = -\frac{b}{a} = -m$

$\alpha^2 - m\beta = 8 \xrightarrow{\alpha^2 = 2m - m\alpha} 2m - m\alpha - m\beta = 8 \to 2m - m(\alpha+\beta) = 8$ ② 7

$\alpha + \beta = ?$

$2m + m^2 = 8 \to m^2 + 2m - 8 = 0$

$(m+4)(m-2) = 0$

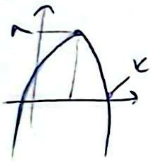
$m = -4$ غ ق ق $\to \Delta < 0$

$m = 2$

$x^2 + 2x - 4 = 0$

$S = -\frac{b}{a} = \boxed{-2}$

$f(x) = mx^2 + 4x + \frac{m}{2} + 7$



$-\frac{\Delta}{4a} = 8 \to \frac{-(16 - 4m(\frac{m}{2}+7))}{4m} = 8 \to \frac{-(16 - 2m^2 - 28m)}{4m} = 8$ ② 8

$2m^2 + 28m - 16 = 32m$

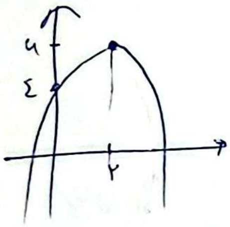
$2m^2 - 4m - 16 = 0$

$m^2 - 2m - 8 = 0 \to (m-4)(m+2) = 0 \ \begin{cases} m = 4 \text{ غ ق ق} \\ m = -2 \ \max \end{cases}$

$\to f(x) = -2x^2 + 4x + 6 = x^2 - 2x - 3 = (x-3)(x+1)$

$\boxed{k = 3}$

$\frac{1}{\alpha} + \frac{1}{\beta} = ? = \frac{\alpha+\beta}{\alpha\beta} = \frac{S}{P}$



$c = 4 \longrightarrow ax^2 + bx + 4$

$\to -\frac{b}{2a} = 2 \to 4a = -b \to 4a + b = 0$

$\xrightarrow{f(2) = 6} 4a + 2b + 4 = 6 \to 4a + 2b = 2$

$b = 2, \quad a = -\frac{1}{2}$

$y = -\frac{1}{2}x^2 + 2x + 4 \leadsto \times(-2): x^2 - 4x - 8 = 0$ ② 9

$\frac{S}{P} \Rightarrow S = 4, \ P = -8 \Rightarrow \frac{4}{-8} = \boxed{-\frac{1}{2}}$

$x^2 - (4a+4)x + (3a^2 + 6a + 3) = 0 \to x^2 - 4(a+1)x + 3(a+1)^2$ ② 10

$\alpha = 2$

$\underline{f(2) = 0} \quad 4 - 8a - 8 + 3a^2 + 6a + 3 = 0$

$3a^2 - 2a - 1 = 0$

$a^2 - 2a - 3 = 0$

$(a - \frac{3}{3})(a + 1) = 0$

$(a-1)(3a+1) = 0$

$a = 1, \quad a = -\frac{1}{3}$

$a = 1: \ x^2 - 8x + 12 \to (x-6)(x-2), \quad x = 6, 2$

$a = -\frac{1}{3}: \ 3x^2 - 8x + 4 \to (x-2)(3x-2), \quad x = 2, \frac{2}{3}$

$\boxed{\beta = 6, \frac{2}{3}}$