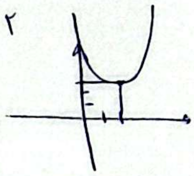
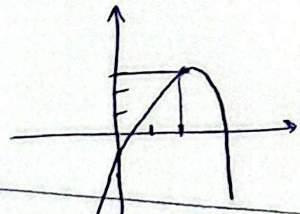
پارمیس عباسیان - تکلیف شماره ۱۲

۱) الف) $\left| \begin{matrix} \frac{-b}{2a} \to 2 \\ 3 \end{matrix} \right.$



ب) $\left| \begin{matrix} \frac{-b}{2a} \to m=2 \\ 3 \end{matrix} \right.$



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۲) الف) $\Delta \geq 0 \to (m+1)^2 - 8(\frac{1}{2}m+2) \geq 0 \to m^2-2m-15>0 \quad (m-5)(m+3)>0$

$m<-3 \cup m>5$

$$\begin{array}{ccccc} & -3 & & 5 & \\ \hline + & | & - & | & + \end{array}$$

ب) $\Delta = 0 \to m^2-2m-15=0 \quad (m-5)(m+3)=0 \to m=\{-3,5\}$

ج) $\Delta<0 \to (m+1)^2 - 8(\frac{1}{2}m+2)<0 \to m^2-2m-15<0 \to \begin{array}{ccccc} & -3 & & 5 & \\ \hline + & | & - & | & + \end{array} \quad -3<m<5$

د) $\Delta \geq 0 \to (m+1)^2 - 8(\frac{1}{2}m+2) \geq 0 \to m^2-2m-15 \geq 0 \quad \begin{array}{ccccc} & -3 & & 5 & \\ \hline + & | & - & | & + \end{array}$

$m \leq -3 \cup m \geq 5$

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۳) الف)

$$\left.\begin{array}{l} \Delta>0 \to 4(m+1)^2-4(m-2)>0 \to 4m^2-12m-84>0 \to m\in\mathbb{R} \\ P>0 \to \frac{10}{m-2}>0 \to \begin{array}{ccc} & 2 & \\ \hline - & | & + \end{array} \to m\in(2,+\infty) \\ S<0 \to \frac{2(m+1)}{m-2}<0 \to \begin{array}{ccccc} & -1 & & 2 & \\ \hline + & | & - & | & + \end{array} \to m\in(-1,2) \end{array}\right\} \varnothing$$

ب)

$$\left\{\begin{array}{l} \Delta>0 \to m\in\mathbb{R} \\ P<0 \quad \begin{array}{ccc} & 2 & \\ \hline - & | & + \end{array} \\ S<0 \to \frac{2m+2}{m-2} \to \begin{array}{ccccc} & -1 & & 2 & \\ \hline + & | & - & | & + \end{array} \to m=(1,2) \end{array}\right.$$

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۴) الف)

$$\left.\begin{array}{l} \Delta>0 \to 4(m+1)^2-4(m-2)>0 \to m\in\mathbb{R} \\ P<0 \to \frac{10}{m-2}<0 \to \begin{array}{ccc} & 2 & \\ \hline - & | & + \end{array} \to m<2 \end{array}\right\} m<2$$

ب) $m \to \frac{-b}{2a}=-2 \to \frac{2(m+1)}{2(m-2)}=-2 \to m=1$

5) $\Delta \geq 0 \quad (2\sin\alpha)^2 - 4\left(\frac{3}{4}\left(\frac{3}{4}\right)\right) \geq 0 \rightarrow 4\sin^2\alpha \geq 4 \rightarrow \sin^2\alpha \geq 1$

$\sin^2\alpha = 1$

$\sin\alpha = \pm 1 \rightarrow \frac{3}{4}m^2 - 2m + \frac{3}{4} = 0 \rightarrow m = \frac{2 \pm \sqrt{0}}{\frac{3}{2}} = \frac{4}{3}$

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6) $-\frac{\Delta}{4a} \longrightarrow \frac{-(3m+1)(-m+1)(2m-5)(m+1)}{(1-m)(1+m)} = -6m^2 + 13m + 5$

$y = \frac{-(13^2 - 4(-6)(5))}{-24} = \frac{1349}{24}$

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7) $3(\alpha+\beta) = 5\alpha\beta + 7 \rightarrow 3\left(\frac{m+2}{m}\right) = -\frac{1}{m} + 7 \rightarrow \frac{3m+6}{m} = \frac{-10+7m}{m}$

$4m = 16 \rightarrow 4 = m$

$S = \alpha + \beta \rightarrow \frac{m+2}{m} \rightarrow \alpha + \beta = \frac{6}{4} = \frac{3}{2}$

$P = \alpha\beta = \frac{-2}{m} \rightarrow \alpha\beta = \frac{-2}{4} = \frac{-1}{2} \rightarrow S^2 - 2P = \alpha^2 + \beta^2 \rightarrow \frac{9}{4} + 1 = \frac{13}{4}$

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8) $\beta^2 = 5\beta - 2$

$\beta^3 = \beta \times \beta^2 = \beta(5\beta - 2) = 5\beta^2 - 2\beta = 5(5\beta - 2) - 2\beta = 25\beta - 10 - 2\beta = 23\beta - 10$

$\beta^4 = \beta \cdot \beta^3 \rightarrow \beta(23\beta - 10) = 23\beta^2 - 10\beta = 23(5\beta - 2) - 10\beta = 105\beta - 46$

$\beta^5 = \beta \cdot \beta^4 \rightarrow \beta(105\beta - 46) = 105\beta^2 - 46\beta = 105(5\beta - 2) - 46\beta$
$= 479\beta - 210$

$\frac{4\alpha + \beta^5}{5\beta^2} = \frac{4(5-\beta) + 479\beta - 210}{5\beta^2} = \frac{475\beta - 190}{5\beta^2} = \frac{95\beta - 38}{\beta^2}$

$\rightarrow \frac{95\beta - 38}{5\beta - 2} = \boxed{19}$

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9) $y = \begin{bmatrix} -3 \\ 4 \end{bmatrix} \rightarrow 4 = 9a - 3b + 4 \rightarrow 3a - b = 0$

$y = \begin{bmatrix} 2 \\ 14 \end{bmatrix} \rightarrow 14 = 4a + 2b + 4 \rightarrow 4a + 4b = 10$

$y = 2m + 10 = \begin{cases} m = 2 \rightarrow y = 14 \\ m = -3 \rightarrow y = 14 \end{cases}$

$\begin{cases} 3a - b = 0 \\ 4a + 4b = 10 \end{cases} \rightarrow \begin{cases} 12a - 4b = 0 \\ 4a + 4b = 10 \end{cases} \Rightarrow 16a = 10 \rightarrow a = \frac{10}{16}$

$b = \frac{15}{8}$

$\frac{10}{16}a^2 - \frac{15}{8}m + 4 = 0$

$m = \frac{-b}{2a} = \frac{3}{2}$

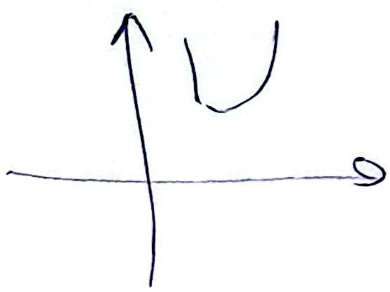
۱۰) $y = x \rightarrow ۲x^۲ + (m+۱)x + m + ۶ = x$

$\rightarrow ۲x^۲ + mx + m + ۶ = ۰$

$\Delta = ۰ \rightarrow m^۲ - ۸(m+۶) = ۰ \rightarrow m^۲ - ۸m - ۴۸ = ۰$

$(m - ۱۲)(m + ۴) = ۰$

$\downarrow$ ۱۲ $\quad \downarrow$ -۴



$a > ۰$

$x = \frac{-b}{۲a} > ۰ \rightarrow b < ۰ \rightarrow m + ۱ < ۰$

$m < -۱$

$\rightarrow m = -۴$ ق ق ✓

$m = ۱۲$ غ ق ق