پرنیان صدرالعاملی — یازدهم ریاضی — تکلیف ۲

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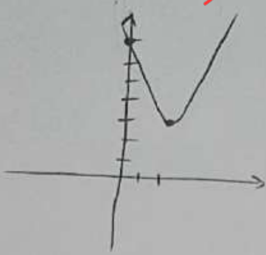
۱- (۲)

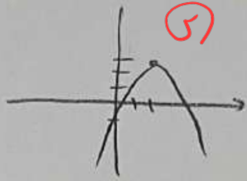
الف) $y = x^2 - 4x + 7$

$$ext \left| \begin{array}{l} \frac{-b}{2a} = \frac{-(-4)}{2} = 2 \\ 4 - 8 + 7 = 3 \end{array} \right.$$



ب) $y = -x^2 + 4x - 1$

$$ext \left| \begin{array}{l} \frac{-b}{2a} = \frac{-4}{-2} = 2 \\ -4 + 8 - 1 = 3 \end{array} \right.$$



۲-

$$2x^2 + (m+1)x + \frac{1}{2}m + 2 = 0$$

الف) دو ریشه متمایز $\Delta > 0$ (۲)

$(m+1)^2 - 4(2)(\frac{1}{2}m+2) > 0 \quad m^2+1+2m-4m-16 > 0 \quad m^2-2m-15 > 0$

$(m-5)(m+3) > 0$ ، $m=5$ ، $m=-3$

	-3		5	
$+$	$\circ$	$-$	$\circ$	$+$

$(-\infty, -3) \cup (5, +\infty)$

ب) ریشه مضاعف $\Delta = 0$

$(m+1)^2 - 4(2)(\frac{1}{2}m+2) = 0 \quad m^2-2m-15 = 0 \quad (m-5)(m+3) = 0$

$m=5$ ، $m=-3$

ج) فاقد ریشه حقیقی $\Delta < 0$

$(m+1)^2 - 4(2)(\frac{1}{2}m+2) < 0 \quad m^2-2m-15 < 0 \quad (m-5)(m+3) < 0$

	-3		5	
$+$	$\circ$	$-$	$\circ$	$+$

$(-3, 5)$

د) دارای ریشه $\Delta \geq 0$

$(m+1)^2 - 4(2)(\frac{1}{2}m+2) \geq 0 \quad m^2-2m-15 \geq 0 \quad (m-5)(m+3)$

	-3		5	
$+$	$\bullet$	$-$	$\bullet$	$+$

$(-\infty, -3] \cup [5, +\infty)$

۳-

$$(m-2)x^2 - 2(m+1)x + 10$$

الف) $S < 0 \quad 0 < P$ (۲)

$S = \frac{2m+2}{m-2} < 0$ ، $m=-1$ ، $m=2$

	-1		2	
$+$	$\circ$	$-$	$\circ$	$+$

$(-1, 2)$

$P = \frac{10}{m-2} > 0$

	2	
$-$	$\circ$	$+$

$(2, +\infty)$

$(2, +\infty) \cap (-1, 2) = \varnothing$

ب) $S < 0 \quad P < 0$

$S = \frac{2m+2}{m-2} < 0$

	-1		2	
$+$	$\circ$	$-$	$\circ$	$+$

$(-1, 2)$

$P = \frac{10}{m-2} < 0$

	2	
$-$		$+$

$(-\infty, 2)$

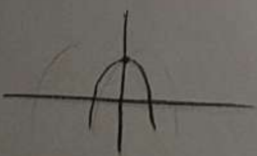
$(-1, 2) \cap (-\infty, 2) = (-1, 2)$

۴-

الف) (۲)

$y = (m-2)x^2 - 2(m+1)x + 10$

عرض از مبدا



$\frac{c}{a} < 0 \quad \frac{10}{m-2} < 0$

	2	
$-$		$+$

$m < 2$

ب) محور تقارن $x = -2$

$\frac{-b}{2a} = -2 \rightarrow \frac{b}{2a} = 2 \qquad \frac{-2(m+1)}{2(m-2)} = 2 \qquad 2m - 4 = -m - 1$

$3m = 3 \quad m = 1$

۵-

$\frac{2x^2}{3} - (2\sin\alpha)x + \frac{3}{2} = 0$ $\qquad \Delta \geq 0 \quad (2\sin\alpha)^2 - 4(\frac{2}{3})(\frac{3}{2}) \geq 0$

$4\sin^2\alpha \geq 4 \quad \sin^2\alpha \geq 1 \quad \sin^2\alpha = 1 \quad \sin\alpha = \pm 1$

$\sin\alpha = 1 \to \frac{2}{3}x^2 - 2x + \frac{3}{2}$ $\qquad \frac{2 \pm \sqrt{4 - 4\times\frac{2}{3}\times\frac{3}{2}}}{\frac{4}{3}} = \frac{6}{4} = \frac{3}{2}$

$\sin\alpha = -1 \to \frac{2}{3}x^2 + 2x + \frac{3}{2}$ $\qquad \frac{-2 \pm \sqrt{4 - 4\times\frac{2}{3}\times\frac{3}{2}}}{\frac{4}{3}} = -\frac{3}{2}$ ققق

۶-

$y = \frac{(3x^2-2x-1)(2x^2-3x-5)}{1-x^2}$

$3-2-1=0 \quad 3x^2-2x-1 \to x_1=1,\ x_2=\frac{c}{a}=\frac{-1}{3}$

$2-5=-3 \quad 2x^2-3x-5 \to x_1=-1,\ x_2=\frac{-c}{a}=\frac{5}{2}$

$\frac{(x-1)(x+\frac{1}{3})(x+1)(x-\frac{5}{2})}{(1-x)(1+x)} = (x+\frac{1}{3})(\frac{5}{2}-x) = \frac{5}{2}x - x^2 + \frac{5}{6} - \frac{1}{3}x = -x^2 + \frac{13}{6}x + \frac{5}{6}$

$y = -\frac{\Delta}{4a} = -\frac{(\frac{13}{6})^2 - 4(-1)(\frac{5}{6})}{-4} = \frac{289}{144}$

۷-

$mx^2 - (m+2)x - 2 = 0$ $\qquad 3S = 5P + 7 \quad 3\frac{(m+2)}{m} = 5(\frac{-2}{m}) + 7$

$\leftarrow \frac{3m+6}{m} = \frac{-10}{m} + 7 = \frac{-10+7m}{m}$

$(3m+6)m = m(-10+7m)$

$3m^2 + 6m = -10m + 7m^2$

$0 = 4m^2 - 16m \quad 4m(m-4) = 0 \quad m \neq 0,\ 4 ✓ \to \alpha^2 + \beta^2 = S^2 - 2P = (\frac{3}{2})^2 + 2(\frac{1}{2}) = \frac{9}{4} + 1 = \frac{13}{4}$

۸- اگر α ، β ریشه‌های معادلهٔ $x^2-5x+2=0$ باشد $\frac{2\alpha+\beta^5}{5\beta^2}$ را بدست آورید.

$S = \alpha + \beta = 5 \Rightarrow \alpha = 5 - \beta$

$\beta^2 - 5\beta + 2 = 0 \quad \beta^2 = 5\beta - 2 \xrightarrow{\times 5} 5\beta^2 = 25\beta - 10$

$\beta^4 = (5\beta-2)^2 = \beta^4 = 25\beta^2 + 4 - 20\beta = 125\beta - 50 + 4 - 20\beta \Rightarrow \beta^4 = 105\beta - 46$

$\xrightarrow{\times\beta} \beta^5 = 105\beta^2 - 46\beta \to 525\beta - 210 - 46\beta = 479\beta - 210 = \beta^5$

$\frac{4(5-\beta) + 479\beta - 210}{5(5\beta-2)} = \frac{20 - 4\beta + 479\beta - 210}{25\beta - 10} = \frac{475\beta - 190}{25\beta - 10} = \frac{19(25\beta-10)}{25\beta-10} = 19$

۹-

$y_1 = ax^2 + bx + c \to 4$

$y_2 = 2x + 10$

$2(2) + 10 = 4a + 2b + 4 \to 2a + b = 5$

$\frac{-6+12}{2} = 9a - 3b + 4$

$2a + b = 5$

$3a - b = 0$

$5a = 5$

$a = 1 \quad b = 3$

$x_S = \text{طول رأس} = \frac{-b}{2a} = \frac{-3}{2}$

$y = 2x^2 + (m+1)x + m + 4$

$\Delta = 0$

$y = 2x^2 + mx + x + m + 4 \rightarrow 2x^2 + mx + m + 4 = 0$

$m^2 - 4(2)(m+4) = 0 \quad m^2 = 8m + 32$

$m^2 - 8m - 48 = 0$

$(m-12)(m+4) = 0$

$m = -4 \rightarrow \Delta = 9 - 4(2)(2) = -\sqrt{32}$ ✓

$m = 12 \rightarrow \Delta = 144 - 4(2)(16) = 16$ ✗

$m = -4$ ✗

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