

الف) $\Delta > 0 \rightarrow b^2 - 4ac > 0 \rightarrow (m+1)^2 - 4(r)(\frac{1}{r}m+r) = m^2 + 2r + 1 - 4m - 4r = m^2 - 2m - 3r = (m-3)(m+r) > 0$

ب) $\Delta = 0 \rightarrow b^2 - 4ac = 0 \rightarrow (m-3)(m+r) = 0 \rightarrow \begin{cases} m=3 \\ m=-r \end{cases}$

ج) $\Delta < 0 \rightarrow b^2 - 4ac < 0 \rightarrow (m-3)(m+r) < 0$

د) $\Delta \geq 0 \rightarrow b^2 - 4ac \geq 0 \rightarrow (m-3)(m+r) \geq 0$

نمودارها:

- الف: $m \mid \begin{matrix} -3 & r \\ + & - \end{matrix}$
- ب: $m \mid \begin{matrix} -r & 3 \\ + & - \end{matrix}$
- د: $m \mid \begin{matrix} -3 & r \\ + & - \end{matrix}$

نتایج:

- الف: $m \in (-\infty, -r) \cup (3, +\infty)$
- ب: $m \in \{3, -r\}$
- د: $m \in (-\infty, -r] \cup [3, +\infty)$

$y = (m-r)x^2 - r(m+1)x + 10$

الف) $\Delta > 0 \rightarrow b^2 - 4ac > 0 \rightarrow r^2(m+1)^2 - 4(m-r)(10) > 0 \rightarrow m^2 - 11m + 11 > 0$

ب) $\Delta > 0 \vee \Delta < 0$

نتایج:

- الف: $m \in (-1, 1) : I$
- ب: $m \in (1, +\infty) : II$

نتیجه نهایی: $I \cap II = \emptyset$

الف) $y = x^2 - rx + v$

ب) $y = -x^2 + rx - 1$

الف: $a > 0, b \min \rightarrow \begin{cases} x_{\min} = \frac{-b}{2a} = \frac{r}{2} \\ y_{\min} = \frac{-4ac}{4a} = \frac{-4(-r)(v)}{4} = rv \end{cases}$

ب: $a < 0, b \max \rightarrow \begin{cases} x_{\max} = \frac{-b}{2a} = \frac{-r}{2(-1)} = \frac{r}{2} \\ y_{\max} = \frac{-4ac}{4a} = \frac{-4(-1)(-1)}{4} = -1 \end{cases}$

نمودارها:

- الف: نمودار سهمی با رأس در $(\frac{r}{2}, rv)$
- ب: نمودار سهمی با رأس در $(\frac{r}{2}, -1)$

$y = (m-r)x^2 - r(m+1)x + 10$

الف) $ac < 0 \rightarrow (m-r)(10) < 0 \rightarrow m < r$

ب) $x = -r, x_{\text{ext}} = \frac{-b}{2a} = -r \rightarrow \frac{r(m+1)}{2(m-r)} = -r \Rightarrow m+1 = -2m+2 \Rightarrow 3m = 1 \Rightarrow m = \frac{1}{3}$

$\frac{r}{r}x^2 - (r \sin \alpha)x + \frac{r}{r} = 0$

الف) $\Delta \geq 0 \rightarrow b^2 - 4ac \geq 0 \rightarrow r^2 \sin^2 \alpha - 4(r)(r) \geq 0 \rightarrow \sin^2 \alpha \geq 1 \rightarrow \begin{cases} \sin \alpha = 1 \\ \sin \alpha = -1 \end{cases} : I$

ب) $\Delta > 0 \rightarrow \frac{b^2}{4a^2} > 0 \rightarrow \frac{r^2 \sin^2 \alpha}{4r^2} > 0 \rightarrow \sin^2 \alpha > 0 \rightarrow \sin \alpha > 0 : II$

پ) $\Delta > 0 \rightarrow \frac{b^2}{4a^2} > 0 \rightarrow \frac{r^2 \sin^2 \alpha}{4r^2} > 0 \rightarrow \sin^2 \alpha > 0 \rightarrow \sin \alpha > 0 : II$

نتیجه نهایی: $I \cap II \Rightarrow \sin \alpha = 1$

نتیجه نهایی: $\sin \alpha = 1 \rightarrow \frac{r}{r}x^2 - rx + \frac{r}{r} = 0 \rightarrow x^2 - rx + 1 = 0 \rightarrow (x-1)^2 = 0 \rightarrow x = 1 \rightarrow x = \frac{r}{r}$

$$r - a = -r \quad \text{و } r - r - 1 = 0 \rightarrow \begin{cases} x = 1 \\ x_1 = \frac{c}{a} = \frac{-1}{r} \end{cases} \rightarrow \begin{cases} x = -1 \\ x_2 = \frac{c}{a} = \frac{a}{r} \end{cases}$$

$$y = \frac{(rx^r - rx - 1)(rx^r - rx - a)}{1 - x^r} = \frac{(x/1)(x + \frac{1}{r})(x/1)(x - \frac{a}{r})}{(x/1)(1 - x)} = \frac{x^r - \frac{1}{r}x - \frac{a}{r}}{-1} =$$

$$\Rightarrow -4x^r + 13x + a = 0$$

$$a < 0 \rightarrow \begin{cases} x_{\max} = \frac{-b}{2a} = \frac{13}{2(-4)} = -\frac{13}{8} \\ y_{\max} = \frac{-a}{4a} = \frac{-b^2 + 4ac}{4a} = \frac{-169 + 4(-4)(a)}{-16} = \frac{169 - 16a}{16} \end{cases}$$

$$y_{\max} = \frac{-a}{4a} = \frac{-b^2 + 4ac}{4a} = \frac{-169 + 4(-4)(a)}{-16} = \frac{169 - 16a}{16}$$

$$mx^r - (m+r)x - r = 0$$

$$rs - ap = v$$

$$\alpha^r + \beta^r = ?$$

$$S = \frac{-b}{a} = \frac{m+r}{m} \xrightarrow{x^r} rs = \frac{r(m+r)}{m}$$

$$P = \frac{c}{a} = \frac{-r}{m} \xrightarrow{x^r} \frac{-10}{m}$$

$$\Rightarrow \frac{r(m+r)+1}{m} = v \Rightarrow r(m+r) = vm \rightarrow r(m+r) = vm \rightarrow m = 6$$

$$\alpha^r + \beta^r = S^r - rP = \frac{9}{r} - r(-\frac{1}{r}) = \frac{9}{r} + 1 = \frac{10}{r}$$

$$\begin{cases} S = \frac{-b}{a} = \frac{r}{r} \\ P = \frac{c}{a} = -\frac{1}{r} \end{cases}$$

$$x^r - ax + r = 0$$

$$S = \frac{-b}{a} = a$$

$$P = \frac{c}{a} = r \rightarrow \alpha\beta = r \rightarrow \alpha = \frac{r}{\beta} \xrightarrow{()^r} \alpha^r = \frac{r}{\beta^r}$$

$$\alpha^r + \beta^r = S^r - rPS = 18a - r(r)(a) = 9a$$

$$\frac{\alpha^r + \beta^r}{\alpha\beta^r} = \frac{1}{a} \left(\frac{\alpha}{\beta} + \beta^a \right) \Rightarrow \frac{1}{a} \times \left(\frac{\alpha}{\beta^r} + \beta^r \right) = \frac{1}{a} (\alpha^r + \beta^r)$$

$$\Rightarrow \frac{1}{a} \times 9a = 19$$

$$\begin{cases} y_1 = ax^r + bx + c \\ y_2 = rx + 10 \end{cases} \Rightarrow \begin{cases} ax^r + bx + c = rx + 10 \\ (x=r, -r) \end{cases} \rightarrow ax^r + (b-r)x - 4 = 0$$

$$\begin{aligned} x = -r &\rightarrow 9a + (-r)(b-r) - 4 = 0 \xrightarrow{x^r} 18a - 4(b-r) - 14 = 0 \\ x = r &\rightarrow 4a + r(b-r) - 4 = 0 \xrightarrow{x^r} 18a + 4(b-r) - 14 = 0 \end{aligned} \Rightarrow \begin{cases} 18a = 18 \\ a = 1 \end{cases}$$

$$a = 1 \rightarrow x + x(b-r) - 4 = 0 \rightarrow b-r = 1 \rightarrow b = r \rightarrow y = x^r + rx + r$$

$$a = 1 \rightarrow b = r, c = r$$

$$y = rx^r + (m+1)x + m+4 \Rightarrow \begin{cases} rx^r + (m+1)x + m+4 = x \\ 0 = 0 \rightarrow b^r - 4ac = 0 \rightarrow m^r - 4(r)(m+4) = 0 \rightarrow m^r - 4m - 16 = 0 \end{cases}$$

$$(m-1)(m+4) = 0 \rightarrow \begin{cases} m = 1 \rightarrow rx^r + 1x + 1 = 0 \rightarrow x^r + 4x + 9 = 0 \rightarrow (x+r)^r = 0 \rightarrow x = -r \\ m = -4 \rightarrow rx^r - 4x + r = 0 \rightarrow x^r - rx + 1 = 0 \rightarrow (x-1)^r = 0 \rightarrow x = 1 \end{cases}$$

$$m = -4$$

$$\begin{cases} x = 1 \\ x > 0 \end{cases}$$