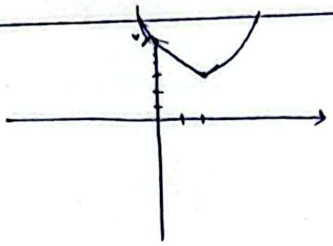
۱-

الف) $y = x^2 - 4x + 7$

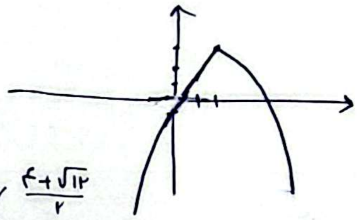
$\left| \begin{array}{l} \frac{4}{2} = 2 \\ 4 - 8 + 7 = 3 \end{array} \right.$



ب) $y = -x^2 + 4x - 1$

$\left| \begin{array}{l} \frac{-4}{-2} = 2 \\ -4 + 8 - 1 = 3 \end{array} \right.$

$$\frac{-4 \pm \sqrt{16-4}}{-2} = \frac{-4 \pm \sqrt{12}}{-2} \begin{cases} \frac{4+\sqrt{12}}{2} \\ \frac{4-\sqrt{12}}{2} \end{cases}$$



۲-

$2x^2 + (m+1)x + \frac{1}{2}m + 2 = 0$

الف) $\Delta > 0$

$(m+1)^2 - 4(2)\underbrace{(\frac{1}{2}m + 2)}_{m+4} > 0$

$m^2 + 1 + 2m - 4m - 16 > 0$

$m^2 - 2m - 15 > 0 \qquad (m-5)(m+3) > 0$

$\begin{array}{c|c|c|c} & -3 & 5 & \\ + & & - & + \end{array}$

$(-\infty, -3) \cup (5, +\infty)$

ب) $\Delta = 0 \qquad m^2 - 2m - 15 = 0$

$(m-5)(m+3) = 0 \quad \boxed{m=5} \quad \boxed{m=-3}$

$2x^2 + 6x + \frac{9}{2} = 0 \qquad 36 - 36 = 0$ ✓

$2x^2 - 2x + \frac{1}{2} = 0 \qquad 4 - 4(2)(\frac{1}{2}) = 0$ ✓

ج) $\Delta < 0$

$m^2 - 2m - 15 < 0$

$(m-5)(m+3) < 0$

$\begin{array}{c|c|c|c} & -3 & 5 & \\ + & & - & + \end{array}$

$(-3, 5)$

د) $\Delta \geq 0$

$m^2 - 2m - 15 \geq 0$

$(m-5)(m+3) \geq 0$

$\begin{array}{c|c|c|c} & -3 & 5 & \\ + & & - & + \end{array}$

$(-\infty, -3] \cup [5, +\infty)$

۳-

$y = (m-2)x^2 - 2(m+1)x + 10$

الف)

$\Delta > 0 \Rightarrow 4\overbrace{(m+1)^2}^{m^2+2m+1} - 4\overbrace{(10)(m-2)}^{10m-20} > 0 \Rightarrow 4m^2 + 8m + 4 - 40m + 80 > 0$

$4m^2 - 32m + 84 > 0$

$m^2 - 8m + 21 > 0$

$64 - 84 < 0$

$\Delta < 0$ همواره مثبت

$S < 0 \quad \frac{2(m+1)}{(m-2)} < 0 \qquad \begin{array}{c|c|c|c} & -1 & 2 & \\ + & & - & + \end{array} \qquad (-1, 2)$

$P > 0 \quad \frac{10}{m-2} > 0 \qquad \begin{array}{c|c|c} & 2 & \\ - & & + \end{array} \qquad (2, +\infty)$

$\xrightarrow{\cap} \varnothing$

ب)

$\Delta > 0 \Rightarrow m^2 - 8m + 21 > 0 \qquad \Delta < 0$ همواره مثبت

$S < 0 \Rightarrow \frac{2(m+1)}{(m-2)} < 0 \qquad \begin{array}{c|c|c|c} & -1 & 2 & \\ + & & - & + \end{array} \qquad (-1, 2)$

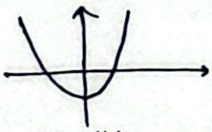
$P < 0 \Rightarrow \frac{10}{m-2} < 0 \qquad \begin{array}{c|c|c} & 2 & \\ - & & + \end{array} \qquad (-\infty, 2)$

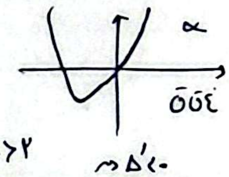
$\xrightarrow{\cap} (-1, 2)$

۴-

$y = (m-2)x^2 - 2(m+1)x + 10$

الف)



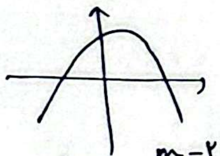


$a > 0 \qquad m - 2 > 0 \Rightarrow m > 2 \qquad \Rightarrow \Delta' < 0$

$\Delta > 0 \Rightarrow 4(m+1)^2 - 4(m-2)(10) > 0 \qquad \Delta > 0$ همواره درست و مثبت

$P < 0 \Rightarrow \frac{10}{m-2} < 0 \qquad \begin{array}{c|c|c} & 2 & \\ - & & + \end{array} \qquad (-\infty, 2)$ ✗

ب)



$a < 0 \qquad m - 2 < 0 \qquad m < 2$

$\Delta > 0 \xrightarrow{\cap}$ همواره مثبت

$P < 0 \qquad \frac{10}{m-2} < 0$

$\begin{array}{c|c|c} & 2 & \\ - & & + \end{array} \qquad (-\infty, 2)$

$\Rightarrow \boxed{(-\infty, 2)}$

ب)

محور تقارن : $x=-2$ $\quad \frac{2(m+1)}{2m-4} = -2 \quad \frac{2m+2}{2m-4} = -2 \Rightarrow 2m+2 = -4m+8$

$6m = 6 \Rightarrow \boxed{m=1}$

5-

$\frac{2x^2}{3} - (2\sin\alpha)x + \frac{3}{2} = 0$

$\sin^2\alpha$ بیشتر از یک نمی‌تواند باشد

$\Delta \geq 0 \Rightarrow 4\sin^2\alpha - \cancel{4}(\frac{2}{\cancel{3}})(\frac{\cancel{3}}{\cancel{2}}) \geq 0 \Rightarrow \sin^2\alpha \geq 1 \Rightarrow \sin^2\alpha = 1 \Rightarrow \sin\alpha = \pm 1$

$\sin\alpha = 1$: $\frac{2x^2}{3} - 2x + \frac{3}{2} = 0 \xrightarrow{\times(6)} 4x^2 - 12x + 9 = 0 \quad (2x-3)(2x-3) = 0 \quad \boxed{x = \frac{3}{2}}$

$\frac{2x^2}{3} + 2x + \frac{3}{2} = 0 \Rightarrow 4x^2 + 12x + 9 = 0 \quad (2x+3)^2 = 0 \Rightarrow x = -\frac{3}{2}$ غ ق ق

6-

$y = \frac{(3x^2-2x-1)(2x^2-3x-5)}{1-x^2} = \frac{6x^4 - 9x^3 - 15x^2 - \cancel{4x^3} + \cancel{6x^2} + 10x - \cancel{2x^2} + 3x + 5}{1-x^2}$

$\frac{6x^4 - 13x^3 - 11x^2 + 13x + 5}{1-x^2}$, $\quad 1-x^2 = -(x^2-1)$

$$\begin{array}{l|l} 6x^4 - 13x^3 - 11x^2 + 13x + 5 & x^2-1 \\ \cancel{-6x^4} + 6x^2 & 6x^2 - 13x - 5 \\ \hline -13x^3 - 5x^2 + 13x + 5 & \\ +13x^3 - 13x & \\ \hline -5x^2 + 5 & \\ +5x^2 - 5 & \\ \hline \end{array}$$

$-\frac{(\cancel{x^2-1})(6x^2-13x-5)}{(\cancel{x^2-1})} = -6x^2 + 13x + 5, y$

$\frac{-\Delta}{4a} = \frac{-(169 - \cancel{4(-6)(5)})}{\cancel{-}24} = \frac{289}{24}$ (120)

7-

$mx^2 - (m+2)x - 2 = 0$

$3(\alpha+\beta) = 5\alpha\beta + 7$

$3\left(\frac{m+2}{m}\right) = 5\left(\frac{-2}{m}\right) + 7 = \frac{3m+6}{m} = \frac{-10}{m} + 7 \Rightarrow \frac{3m+16}{m} = 7 \Rightarrow 3m+16 = 7m$

$4m = 16 \Rightarrow \boxed{m=4}$

$\alpha^2 + \beta^2 = S^2 - 2P = \frac{9}{4} - 2(-\frac{1}{2}) = \boxed{\frac{13}{4}}$

$S = \frac{6}{4} = \frac{3}{2} \quad \frac{-2}{4} = -\frac{1}{2}$

8-

$x^2 - 5x + 2 = 0 \quad P = 2 \quad \alpha\beta = 2 \Rightarrow \beta = \frac{2}{\alpha} \Rightarrow \beta^2 = \frac{4}{\alpha^2}$

$\frac{4\alpha + \beta^5}{5\beta^2} = \frac{4\alpha}{5\beta^2} + \frac{\beta^3}{5} \Rightarrow \frac{\cancel{4}\alpha}{5(\frac{\cancel{4}}{\alpha^2})} + \frac{\beta^3}{5} = \frac{\alpha^3}{5} + \frac{\beta^3}{5} = \frac{\alpha^3+\beta^3}{5} = \frac{95}{5} = \boxed{19}$

$\alpha^3 + \beta^3 = S^3 - 3SP \Rightarrow 125 - \underbrace{3(5)(2)}_{30} = 95$

9-

$y_1 = ax^2 + bx + c \quad c = 4$

$y_2 = 2x + 10 \qquad y = x^2 + 3x + 4$

$\xrightarrow{x=2} 4a + 2b + 4 = 14 \Rightarrow 12a + 6b = 30$ $\quad 2b = 6 \quad b = 3$

$\xrightarrow{x=-3} 9a - 3b + \cancel{4} = \cancel{4} \Rightarrow 18a - 6b = 0$

$30a = 30 \quad \boxed{a=1}$

$-\frac{3}{2} \quad x = -\frac{3}{2}$

10-

$y = 2x^2 + (m+1)x + m + 6 \qquad 2x^2 + (m+1)x + m + 6 = x$

$y = x \qquad \Rightarrow 2x^2 + mx + m + 6 = 0$ مماس است $\Delta = 0$

$m^2 - 4(2)(m+6) = 0$

$m^2 - 8m - 48 = 0$

$(m-12)(m+4) = 0$

$\boxed{m=12}$ ✓ $\quad 2x^2 + 12x + 18 = 0 \quad \Delta = 0$

$\boxed{m=-4}$ ✓ $\quad 2x^2 - 4x + 2 = 0 \quad \Delta = 0$