بهار نیازمهی / تکلیف ۱۲ / یازدهم دختران c /

به نام خدا

۱)

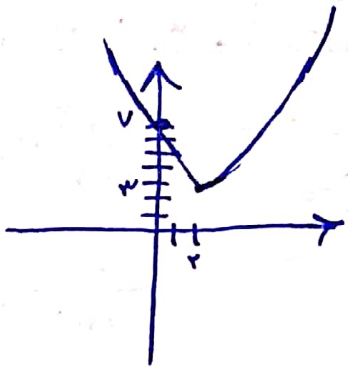
الف)

ext | $-\frac{b}{2a} = \frac{4}{2} = 2$; 3

x	0	2
y	7	3

$y = x^2 - 4x + 7$

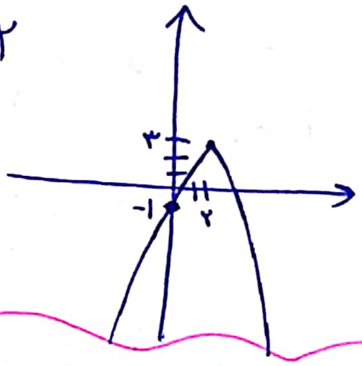
$\hookrightarrow \Delta < 0$



ب) $y = -x^2 + 4x - 1$

ext | $-\frac{b}{2a} = \frac{-4}{-2} = 2$; 3

x	0	2
y	-1	3



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۲) $2x^2 + (m+1)x + \frac{1}{2}m + 2 = 0$ مقدار m ؟

الف) معادله ۲ ریشه متمایز داشته باشد.

الف) $\Delta > 0 \to (m+1)^2 - (4 \times (\frac{1}{2}m + 2) \times 2) > 0 \to m^2 + 1 + 2m - 4m - 16 > 0$

$m^2 - 2m - 15 > 0 \to (m-5)(m+3) > 0 \to$

	-3		5	
+	0	-	0	+

$\hookrightarrow m \in (-\infty, -3) \cup (5, +\infty)$

ب) $\Delta = 0 \to m^2 - 2m - 15 = 0 \to m = 5 , m = -3$

ج) $\Delta < 0 \to m^2 - 2m - 15 < 0 \to (m-5)(m+3) < 0 \to$

	-3		5	
+	0	-	0	+

$m \in (-3, 5)$

د) $\Delta \geq 0 \to (m-5)(m+3) \geq 0 \to$

	-3		5	
+	0	-	0	+

$\to m \in (-\infty, -3] \cup [5, +\infty)$

③

$y=(m-2)x^2-2(m+1)x+10$

الف) $\Delta>0 \rightarrow 4m^2-32m+84>0 \rightarrow$ ر لتای اینجا همیشه درست است

$S<0 \rightarrow \frac{2m+2}{m-2}<0 \rightarrow$ $\underset{+\ \ -\ \ +}{-1\quad 2}$ $\Rightarrow (-1,2)$

$P>0 \rightarrow \frac{10}{m-2}>0 \rightarrow$ $\underset{-\ \ +}{2}$ $\Rightarrow (2,+\infty)$

$\xRightarrow{\cap} \varnothing$

ب) $\Delta>0 \rightarrow$ همواره درست

$S<0 \rightarrow \frac{10}{m-2}<0$ $\underset{-\ \ +}{2}$ $\rightarrow (-\infty,2)$

$P<0 \rightarrow \frac{2m+2}{m-2}<0 \rightarrow$ $\underset{+\ \ -\ \ +}{-1\quad 2}$ $\rightarrow (-1,2)$

$\xRightarrow{\cap} (-1,2)$

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④

$y=(m-2)x^2-2(m+1)x+10$

الف)

Max $\begin{cases} \Delta>0 \rightarrow \checkmark \\ m-2<0 \rightarrow m<2 \\ P<0 \rightarrow \frac{10}{m-2}<0 \rightarrow \underset{-\ \ +}{2} \quad (-\infty,2) \end{cases}$ $\xrightarrow{\cap} (-\infty,2)$

min $\begin{cases} m-2>0 \rightarrow m>2 \\ \Delta>0 \ \checkmark \\ P<0 \rightarrow \frac{10}{m-2}<0 \quad \underset{-\ \ +}{2} \rightarrow (-\infty,2) \end{cases}$ $\xrightarrow{\cap} \varnothing$

$(-\infty,2)$

ب) $\frac{-b}{2a}=-2 \rightarrow \frac{2m+2}{2m-4}=-2 \rightarrow 2m+2=-4m+8 \rightarrow 6m=6 \rightarrow m=1$

(5)

$$\frac{2x^2}{3} - (2\sin\alpha)x + \frac{3}{2} = 0$$

$$\Delta \geq 0 \rightarrow 4\sin^2\alpha - 4\cdot\frac{2}{3}\cdot\frac{3}{2} \geq 0 \Rightarrow \sin^2\alpha \geq 1 \rightarrow \sin^2\alpha = 1$$

$$\sin\alpha = \pm 1 \xrightarrow{+1} \frac{2}{3}x^2 - 2x + \frac{3}{2} = 0 \xrightarrow{\times 12} 8x^2 - 24x + 9 = 0 \quad (*)$$

$$\xrightarrow{-1} \frac{2}{3}x^2 + 2x + \frac{3}{2} = 0 \rightarrow x = -12/\bar{8}\bar{8}\bar{4}$$

$$(*) \xrightarrow{m=1,2} x^2 - 12x + 32 = 0 \rightarrow (x-8)(x-4) = 0 \Rightarrow x = 8, 4$$

$$\boxed{x = \frac{3}{2}}$$

(6)

$$\frac{(3x^2 - 2x - 1)(2x^2 - 3x - 5)}{1 - x^2} = -\frac{6x^4 - 13x^3 - 11x^2 + 13x + 5}{(x-1)(x+1)} \quad x \neq \pm 1$$

$$6x^4 - 13x^3 - 11x^2 + 13x + 5 \;\big|\; x^2 - 1$$

$$-6x^2 + 13x + 5$$

$$y = -6x^2 + 13x + 5 \rightarrow x = -\frac{b}{2a} = \frac{-13}{-12} = \frac{13}{12}, \; y = \frac{289}{24}$$

(7)

$$mx^2 - (m+2)x - 2 = 0$$

$$3(\alpha+\beta) = 5(\alpha\beta) + 7 \rightarrow \frac{3m+6}{m} = \frac{-10}{m} + 7 \rightarrow \frac{3m+16}{m} = 7$$

$$\frac{m+2}{m} \qquad -2/m$$

$$3m + 16 - 7m = 0 \rightarrow 4m = 16 \rightarrow m = 4$$

$$4x^2 - 6x - 2 = 0$$

$$\rho = -\frac{2}{m} = -1/2$$

$$\sigma = \frac{m+2}{m} = 6/4$$

$$\hookrightarrow \alpha^2 + \beta^2 = \sigma^2 - 2\rho$$

$$= 9/4 + 4/4 = \boxed{13/4}$$

۱) $x^2 - 5x + 2 = 0$ $\quad \frac{4\alpha}{5\beta^2} + \frac{\beta^3}{5} = ?$

$\alpha\beta = 2 \to \beta = \frac{2}{\alpha}$ $\quad \alpha + \beta = 5$

$\frac{4\alpha}{5 \times \frac{4}{\alpha^2}} = \frac{\alpha^3}{5} \Rightarrow \frac{\alpha^3 + \beta^3}{5} = \frac{5^3 - 3 \times 2 \times 5}{5} = \frac{125 - 30}{5} = 19$

۲) $y_1 = ax^2 + bx + c$, $y_2 = 2x + 10$ $\to$ $y_2 = 2 \times 2 + b = 14$ $\quad (2,0)(-3,0)(0,4)$

$y_1 = 4a + 2b + 4 \to 14 = 4a + 2b + 4$

$10 = 2(2a + b) \Rightarrow 2a + b = 5$ (I)

$y_1 = 9a - 3b + 4$, $y_2 = -2 + 10 = 8$ $\to$ $9a - 3b = 0 \to 3(3a - b) = 0 \to 3a - b = 0$ (II)

(I), (II) $\Rightarrow$ $\begin{cases} 2a + b = 5 \\ 3a - b = 0 \end{cases}$

$5a = 5 \to a = 1, b = 3$

$y_1 = x^2 + 3x + 4 \to x = -\frac{b}{2a} = -\frac{3}{2}$

۱۵) $y = 2x^2 + (m+1)x + m + 4$ $\quad$ نیم‌ساز ناحیه اول $\to x = y$

$2x^2 + (m+1)x + m + 4 = x$

$2x^2 + (m)x + m + 4 = 0$ $\quad \Delta < 0 \to m^2 - 8m - 48 < 0$

$(m+4)(m-12) < 0$

-4 $\quad$ 12

✓ $a > 0 \to$ سهمی رو به بالا

if $m = 12 \to y = 2x^2 + 12x + 18 \to \Delta > 0$

if $m = -4 \to y = 2x^2 - 3x + 3 \to \Delta < 0 \to$ جواب $m = -4$