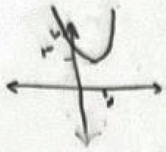
۱)

$y = x^2 - 4x + 7 \quad \left| \begin{array}{l} \frac{-b}{2a} = \frac{4}{2} = 2 \\ 4 - 8 + 7 = 3 \end{array} \right.$



$y = -x^2 + 4x - 1 \quad \left| \begin{array}{l} \frac{-b}{2a} = \frac{-4}{-2} = 2 \\ -4 + 8 - 1 = 3 \end{array} \right.$

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۲)

$2x^2 + (m+1)x + \frac{1}{2}m + 2 = 0$

الف) $\Delta > 0 \to (m+1)^2 - 4\underbrace{(2)(\frac{1}{2}m+2)}_{m+4} \geq 0 \to m^2 + 1 + 2m - 4m - 16 \geq 0$

$m^2 - 2m - 15 \geq 0 \to (m-5)(m+3) \geq 0$ $\quad \begin{array}{ccccc} & -3 & & 5 & \\ + & | & - & | & + \end{array} \to m \in (-\infty, -3) \cup (5, +\infty)$

ب) $\Delta = 0 \to (m+1)^2 - 4(2)(\frac{1}{2}m+2) = 0 \to m^2 - 2m - 15 = 0 \to (m-5)(m+3) = 0$

$m = 5, \ m = -3$

ج) $\Delta < 0 \to (m+1)^2 - 4(2)(\frac{1}{2}m+2) < 0 \to m^2 - 2m - 15 < 0 \to \begin{array}{ccccc} & -3 & & 5 & \\ + & | & - & | & + \end{array}$ $\boxed{m \in (-3, 5)}$

د) $\Delta \geq 0 \to (m+1)^2 - 4(2)(\frac{1}{2}m+2) \geq 0 \to m^2 - 2m - 15 \geq 0 \to (m-5)(m+3)$

$\begin{array}{ccccc} & -3 & & 5 & \\ + & | & - & | & + \end{array} \to m \in (-\infty, -3] \cup [5, +\infty)$

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۳)

$y = (m-2)x^2 - 2(m+1)x + 10$

الف) $\Delta \geq 0 \to (-2m-2)^2 - 4(10)(m-2) \to 4m^2 + 4 + 8m - 40m + 80 \geq 0$

$4m^2 - 32m + 84 \geq 0 \quad m^2 - 8m + 21 \geq 0 \to \Delta < 0$

$S < 0 \to \frac{2m+2}{m-2} < 0 \quad \begin{array}{ccccc} & -1 & & 2 & \\ + & | & - & | & + \end{array} \quad (-1, 2)$

$P > 0 \quad \frac{10}{m-2} > 0 \quad \begin{array}{ccc} & 2 & \\ - & | & + \end{array} \to m > 2$

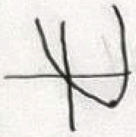
ب)

$\frac{c}{a}=\frac{10}{m-2}<0 \quad \begin{array}{c|c} & 2 \\ - & + \end{array} \to m<2$

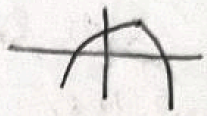
$\frac{2m+2}{m-2}<0 \quad \begin{array}{c|c|c} & -1 & 2 \\ + & - & + \end{array} \to -1<m<2$

$\xrightarrow{\cap} -1<m<2$

۴) $y=(m-2)x^2-2(m+1)x+10$



$(m-2)x^2-2(m+1)x+10$



$\Delta>0 \to 4(m+1)^2-4(10)(m-2)>0$ ✓ $\quad \Delta>0$ ✓

$a>0 \quad m-2>0 \quad m>2$ $\quad \xrightarrow{\cap} \varnothing$ $\quad a<0 \quad m-2<0 \quad m<2$

$c<0 \quad \frac{10}{m-2}<0 \quad \begin{array}{c|c} & 2 \\ - & + \end{array} \to m<2$ $\quad c<0 \quad \frac{10}{m-2}<0 \quad m<2 \xrightarrow{\cap} \boxed{m<2}$

۵) $\Delta\geq0 \to 4\sin^2\alpha-4(\frac{2}{3})(\frac{3}{2})\geq0 \quad 4\sin^2\alpha-4\geq0 \to 4\sin^2\alpha\geq4 \to \sin^2\alpha\geq1$

$\sin^2\geq1 \to \sin^2\alpha=1 \quad \sin\alpha=\pm1$

$\frac{2}{3}x^2+2x+\frac{3}{2}=0 \quad 4x^2+12x+9=0 \quad (2x+3)^2=0 \quad x=-\frac{3}{2}$ ✗

$2x^2-2x+\frac{3}{2}=0 \to 4x^2-12x+9=0 \quad (2x-3)^2=0 \quad x=\frac{3}{2}$ ✓

۶) $y=\frac{(3x^2-2x-1)(2x^2-3x-5)}{1-x^2}=\frac{(x+\frac{1}{3})\cancel{(x-1)}\cancel{(x+1)}(x-\frac{5}{2})}{\cancel{(1-x)}\cancel{(1+x)}} \to y=x^2-\frac{13}{6}x-\frac{5}{6}$

$x\neq1, x\neq-1$

$ax^2+bx+c \to a+b+c \to x=\frac{c}{a}=1 \quad ax^2+bx+c \to a+c=b \to x=-\frac{c}{a}=-1$

$y=6x^2-13x-5$

$\frac{-\Delta}{4a}=\frac{b^2-4ac}{4a}=\frac{-289}{24}$

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۷)

$3S = 5P + 7 \to 3\left(\frac{m+2}{m}\right) = 5\left(\frac{-2}{m}\right) + 7 \to \frac{3m+6}{m} = \frac{-10}{m} + 7$

$\frac{3m+6+10-7m}{m} = 0 \to -4m + 16 = 0 \to -4m = -16 \to \boxed{m=4}$

۸)

$\frac{4\alpha + \beta^5}{5\beta^2} = \frac{4\alpha}{5\beta^2} + \frac{\beta^3}{5} \to \frac{4\alpha}{5\left(\frac{2}{\alpha^2}\right)} + \frac{\beta^3}{5} = \frac{\alpha^3+\beta^3}{5} = \frac{S^3-3PS}{5} = \frac{95}{5} = 19$

$S = 5 \quad P = 2$

۹)

$y_1 = ax^2 + bx + 4 \xrightarrow{x=2} 4a+2b+4 = 14 \to 4a+2b = 10 \to 2a+b = 5$

$\xrightarrow{x=-3} 9a-3b+4 = 4 \to 9a-3b = 0 \to 3a-b = 0$

$5a = 5 \quad \boxed{a=1} \quad \boxed{b=3}$

$y = x^2+3x+4 \to \frac{-b}{2a} = \boxed{\frac{-3}{2}}$

۱۰)

$y = 2x^2 + (m+1)x + m + 6$ و $x = y \to 2x^2 + (m+1)x + m + 6 = x \to$

$2x^2 + mx + m + 6 = 0 \to m^2 - 4(2)(m+6) = 0 \quad m^2 - 8m - 48 = 0 \quad (m-12)(m+4)$

$m = 12, \; m = -4$