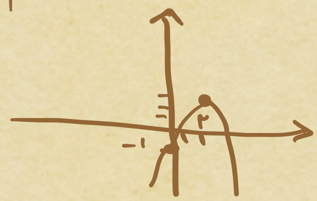
تکلیف ۱۲ - ۱۱ دهم c

سهامهادی

(۱)

الف) $y = x^2 - 4x + 7 \to \min \left| \begin{array}{l} \frac{-b}{2a} = 2 \\ 4 - 8 + 7 = 3 \end{array} \right.$

ب) $y = -x^2 + 4x - 1 \quad \max \left| \begin{array}{l} \frac{-b}{2a} = 2 \\ -4 + 8 - 1 = 3 \end{array} \right.$



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(۲) الف) $y = \underbrace{2x^2 + (m+1)x + \frac{1}{2}m + 2 = 0}$ $\to \Delta > 0 \to (m+1)^2 - 4 \times 2 \times (\frac{1}{2}m + 2) > 0$

$\to m^2 + 2m + 1 - 4m - 16 > 0$

$m^2 - 2m - 15 > 0$

$$\begin{array}{ccccc} & -3 & & 5 & \\ \hline + & 0 & - & 0 & + \end{array}$$

$m \in (-\infty, -3) \cup (5, +\infty)$

ب) $\Delta = 0 \to m^2 - 2m - 15 = 0 \to (m-5)(m+3) = 0 \to m = -3, 5$

ج) $\Delta < 0 \to m^2 - 2m - 15 < 0$ $\quad \begin{array}{ccccc} & -3 & & 5 & \\ \hline + & 0 & - & 0 & + \end{array}$ $\to m \in (-3, 5)$

د) $\Delta \geq 0 \to m^2 - 2m - 15 \geq 0$ $\quad \begin{array}{ccccc} & -3 & & 5 & \\ \hline + & 0 & - & 0 & + \end{array}$ $\to m \in (-\infty, -3] \cup [5, +\infty)$

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(۳) $y = (m-2)x^2 - 2(m+1)x + 10$

الف) $\Delta > 0 \to (2m+2)^2 - 4 \times 10 \times (m-2) > 0 \to 4m^2 - 32m + 84 > 0$

$\div 4 \Rightarrow m^2 - 8m + 21 > 0$ همواره برقرار

$\to P > 0 \to \frac{10}{m-2} > 0 \to m > 2$ (1)

$S < 0 \to \frac{2m+2}{m-2} < 0$ $\quad \begin{array}{ccccc} & -1 & & 2 & \\ \hline + & 0 & - & \nexists & + \end{array}$ $\to -1 < m < 2$ (2)

$(1) \cap (2) = \varnothing$

ب)

$S < 0 \to \frac{2m+2}{m-2} < 0$ $\quad \begin{array}{ccccc} & -1 & & 2 & \\ \hline + & 0 & - & \nexists & + \end{array}$ $\to -1 < m < 2$

$P < 0 \to \frac{10}{m-2} < 0 \to m < 2$

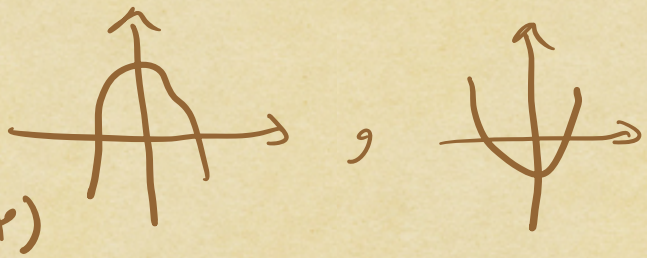
$m \in (-1, 2)$

(۴)

$y = (m-۲)x^۲ - ۲(m+۱)x + ۱۰$

(الف)

$P < ۰ \Rightarrow \frac{۱۰}{m-۲} < ۰ \rightarrow m-۲ < ۰ \leftarrow$



$\rightarrow m < ۲ \rightarrow \underline{m \in (-\infty, ۲)}$

(ب)

$\frac{-b}{۲a} = \frac{۲m+۲}{۴m-۴} = -۴ \Rightarrow -۴m+۸ = ۲m+۲$

$c = c_m \rightarrow \underline{m = ۱}$

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(۵)

$\frac{۲x^۲}{۳} - (۲\sin\alpha)x + \frac{۳}{۲} = ۰ \rightarrow \Delta \geq ۰ \Rightarrow ۴\sin^۲\alpha - ۴ \times \frac{۲}{۳} \times \frac{۳}{۲} \geq ۰$

$\Rightarrow ۴\sin^۲\alpha - ۴ \geq ۰$

$\rightarrow ۴\sin\alpha \geq ۴ \rightarrow \sin^۲\alpha \geq ۱ \xrightarrow{۰ \leq \sin^۲\alpha \leq ۱} \sin^۲\alpha = ۱ \rightarrow \sin = \pm ۱$

$\begin{cases} \sin = ۱ \rightarrow \frac{۲}{۳}x^۲ - ۲x + \frac{۳}{۲} = ۰ \rightarrow \Delta = ۴ - ۴ \times \frac{۲}{۳} \times \frac{۳}{۲} = ۰ \\ \sin = -۱ \rightarrow \frac{۲}{۳}x^۲ + ۲x + \frac{۳}{۲} = ۰ \rightarrow \Delta = ۰ \end{cases}$

$\rightarrow x = \frac{۲ \pm ۰}{۲ \times \frac{۲}{۳}} = ۳$ (۳)

$\rightarrow x = \frac{-۲ \pm ۰}{۴ \times \frac{۲}{۳}} \rightarrow -\frac{۳}{۴}$ (۴)

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(۶)

$y = \frac{(۳x^۲ - ۲x - ۱)(۴x^۲ - ۳x - ۵)}{۱ - x^۲}$ $\rightarrow a+c=b \rightarrow x = -۱, \frac{۵}{۴}$

$a+b+c=۰ \rightarrow x = ۱, -\frac{۱}{۳} \rightarrow \frac{(x-۱)(۳x+۱)(x+۱)(۴x-۵)}{(۱-x)(۱+x)}$

$\rightarrow (۳x+۱)(۴x-۵) = ۱۲x^۲ - ۱۱x - ۵ \rightarrow y = \frac{-\Delta}{۴a} = \frac{-۱۲۱}{۴۸}$

$mx^2-(m+2)x-2=0 \rightarrow S=\alpha+\beta \rightarrow \frac{m+2}{m}=\frac{-2}{m} \times \alpha+1$ (۷)

$\rightarrow \frac{3m+4}{m}=\frac{7m-10}{m}$

$\rightarrow 3m+4=7m-10 \rightarrow 4m=14 \rightarrow m=4$

$\alpha^2+\beta^2=S^2-2P=\left(\frac{3m+4}{m}\right)^2-2\left(\frac{-2}{m}\right)=\frac{9}{4}+1=\frac{13}{4}$

$x^2-5x+2=0$ (۸)

$S=5 \rightarrow \alpha=5-\beta$

$P=2$

$\rightarrow \beta^2=5\beta-2$

$\frac{4\alpha+\beta^5}{5\beta^2}=$

$\beta^4=(5\beta-2)^2=25\beta^2-20\beta+4 \rightarrow 25(5\beta-2)-20\beta+4=0$

$125\beta-50-20\beta+4=0$

$\rightarrow 105\beta=46 \xrightarrow{\times\beta}$

$\beta^5=105\beta^2-46\beta$

$\leftarrow \beta^2=5\beta-2$

$\beta^5=479\beta-210$

$\rightarrow \frac{4(5-\beta)+479\beta-210}{5(5\beta-2)}=\frac{475\beta-190}{25\beta-10}=\frac{19(25\beta-10)}{25\beta-10}$

$=19$

$y_1=ax^2+bx+c \xrightarrow{(0,4)} c=4$ (۹)

$y_2=2x+10 \xrightarrow{x=2} y=4 \rightarrow 14=4a+2b+4 \rightarrow 2a+b=5$

$x=-3 \rightarrow y=4 \rightarrow 4=9a-3b+4 \rightarrow 9a=3b \rightarrow b=3a$

$\rightarrow 2a+3a=5 \rightarrow a=1, b=3$

$y = 2x^2 + (m+1)x + m + 4 = x \rightarrow 2x^2 + mx + m + 4 = 0$ (10)

$\Delta = 0 \Rightarrow m^2 - 4 \times 2 \times (m+4) = 0$

$m^2 - 8m - 32 = 0$

$(m-12)(m+4)$ $\begin{cases} m = 12 \\ m = -4 \end{cases}$