$$f(x)=\begin{cases} x^3-4, & x>a \rightarrow a^3-4 \\ 12x-20, & x\leq a \rightarrow 12a-20 \end{cases}$$

$a^3-4 \geq 12a-20$

$\rightarrow a^3-12a+16 \geq 0$

$(a-2)(a^2+2a-8) \geq 0 \quad (a+4)(a-2)$

$(a-2)(a+4)(a-2) \geq 0$

	-4		2	
$-$		$+$		$+$

$\rightarrow a \geq -4$

---

۲. الف) $f(x)=3x+k \ , \ f^{-1}(2)=4$

$f^{-1}(x)=x=3y+k \rightarrow 3y=x-k \rightarrow y=\frac{2-k}{3}=4 \rightarrow \boxed{k=-10}$

$f(7)=21-10=11$

ب) $f(f(x))=9x-40$

$3(3x-10)-10=9x-30-10 \rightarrow 9x-40$

---

۳. $f(x)=\frac{ax}{x-1} \rightarrow f^{-1}(x)=x=\frac{ya}{y-1}=y=\frac{x}{x-a}$

$A=\begin{bmatrix} 2a \\ a \end{bmatrix} \rightarrow \frac{2a}{2a-a}=a \Rightarrow \boxed{a=2}$

---

۴. $fof^{-1}=\{(5,5),(3,3),(7,7),(6,6)\}$

$f^{-1}of=\{(3,3),(4,4),(7,7),(9,9)\}$

$fog^{-1}=\{(3,5),(5,4)\}$

$g^{-1}of=\{(2,9),(7,2),(9,8)\}$

5. $f=\{(2,4),(4,8),(6,0)\}$ $\qquad fog^{-1}=\{(1,4),(3,8),(5,0)\}$

$g=\{(2,1),(4,3),(6,5)\}$

$f=\{(2,4),(4,8),(6,0)\}$

$\frac{h}{fog^{-1}} \rightsquigarrow \{(1,\frac{1}{2}),(3,\frac{1}{2}),\cancel{\ }\} \to \{(1,\frac{1}{2}),(3,\frac{1}{2})\}$

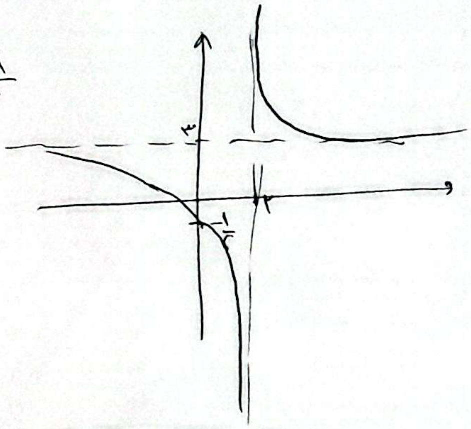
---

6. $y=\frac{3x+1}{x-2}$

$\frac{a}{c}=3$

$x=2$

x	y
0	$-\frac{1}{2}$

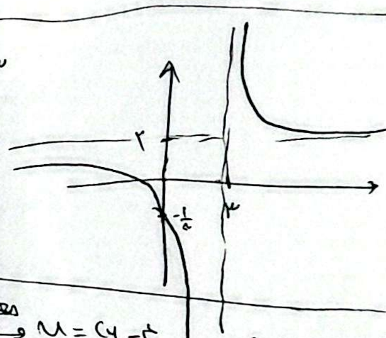


$\frac{3y+1}{y-2}=x \to y=\frac{2x+1}{x-3}$

$x=3$

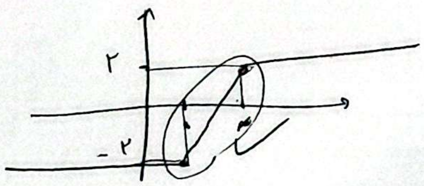
$\frac{a}{c}=2$

x	y
0	$-\frac{1}{3}$



---

7. $f(x)=|x-1|-|x-3|$ ① ③



$y=ax+b$

$\begin{cases}3a+b=2\\-a-b=2\end{cases} \to \begin{cases}2a=4\\ \boxed{a=2}\ \boxed{b=-4}\end{cases} \to y=2x-4$

$y=2x-4 \to x=2y-4 \to 2y=x+4$

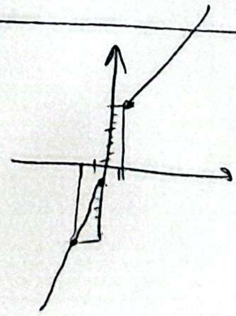
$\rightsquigarrow y=\frac{x+4}{2}$

$f^{-1}(x)=\frac{x+4}{2}$

$R_{f(x)} \to [-2,2]$

---

8. $f(x)=\begin{cases}x^3+4 & x\geq 1 \to R=[5,+\infty)\\ 4x-1 & x\leq 0 \to R=[-5,1]\end{cases}$



$x=y^3+4 \to y^3=x-4 \to y=\sqrt[3]{x-4}$

$x=4y-1 \to 4y=x+1 \to y=\frac{x+1}{4}$

$f^{-1}(x)=\begin{cases}\sqrt[3]{x-4} & x\geq 5\\ \frac{x+1}{4} & x\leq -1\end{cases}$

۹. $f(x) = x^2 - \frac{(x+1)^2}{x+2} \rightarrow f(x) = \frac{x^3 + 2x^2 - x^2 - 1 - 2x^2 - 2x}{x+2}$

$\Rightarrow \frac{-2x-1}{x+2} \rightarrow y = \frac{-2x-1}{x+2} \rightarrow x = \frac{-2y-1}{y+2}$

$y = \frac{-2x-1}{x+2} \rightarrow \frac{ax+b}{2x+d}$ $\begin{matrix} a=-1 \\ b=-2 \\ d=1 \end{matrix}$

$f^{-1}(b) = f^{-1}(-2) = \frac{-1(-2)-1}{-2+2} = 0$

---

۱۰. $f(x) = \frac{x}{x^2+1} \rightarrow y(x^2+1) = x$

$yx^2 + y = x \rightarrow yx^2 + y - x = 0$

$\rightarrow x = \frac{-b \pm \sqrt{\Delta}}{2a} \rightarrow a = y \quad b = -1 \quad c = y$

$x = \frac{-(-1) \pm \sqrt{1-4y^2}}{2y} \rightarrow f^{-1}(x) = \frac{1+\sqrt{1-4x^2}}{2x}$

$1-4x^2 \geq 0$

$4x^2 \leq 1 \quad x^2 \leq \frac{1}{4}$

$\rightarrow \boxed{x \leq \frac{1}{2}}$

$2x \rightarrow x \neq 0$

$\rightarrow (0, \frac{1}{2}]$