

$$f(u) = \begin{cases} u^2 - 2 & ; u > a \\ 12u - 20 & ; u \leq a \end{cases}$$

$$u^2 - 2 = 12u - 20 \rightarrow u^2 - 12u + 18 = 0 \Rightarrow (u-2)^2 (u+2) \geq 0 \Rightarrow \boxed{a \geq -2}$$

که جواب

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$$f^{-1}(2) = 2 \rightarrow f(a) = b \Leftrightarrow f^{-1}(b) = a \Rightarrow f(2) = 2$$

$$f(u) = 3u + k \Rightarrow f(2) = 2 \Rightarrow f(2) = 3 \times 2 + k \Rightarrow 2 = 6 + k \Rightarrow \boxed{k = -4}$$

$$\hookrightarrow f(u) = 3u - 4$$

$$الف) f(u) = 3 \times 10 - 4 = 26 \rightarrow \text{جواب}$$

$$ب) f(f(u)) = 3(3u - 4) - 4 = 9u - 12 - 4 = 9u - 16 \rightarrow \text{جواب}$$

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$$f(u) = \frac{au}{u-1} \Rightarrow y = \frac{au}{u-1} \xrightarrow{\text{حاصل‌گیری}} yu - y = au \Rightarrow yu - au = y \Rightarrow u(y-a) = y$$

$$\Rightarrow u = \frac{y}{y-a} \Rightarrow f^{-1}(u) = \frac{u}{u-a}$$

$$\hookrightarrow A(2a, a) \Rightarrow a = \frac{2a}{2a-a} \Rightarrow a = \frac{2a}{a} \Rightarrow \boxed{a=2}$$

که جواب

* نکته: A روی این تابع پس مشخص
آنی باید در تابع صدق کند

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$$g = \{(2, 4), (1, 2), (9, 5)\} \Rightarrow g^{-1} = \{(4, 2), (2, 1), (5, 9)\} \text{ و } f = \{(3, 5), (2, 1), (7, 2), (9, 4)\} \Rightarrow f^{-1} = \{(5, 3), (1, 2), (2, 7), (4, 9)\}$$

$$الف) f \circ f^{-1} = f(f^{-1}(u)) = \{(5, 5), (1, 1), (2, 2), (4, 4)\} \Rightarrow \text{جواب}$$

$$ب) f^{-1} \circ f = f^{-1}(f(u)) = \{(3, 3), (2, 2), (7, 7), (9, 9)\} \Rightarrow \text{جواب}$$

$$ج) f \circ g^{-1} = f(g^{-1}(u)) = \{(2, 5), (5, 4)\} \Rightarrow \text{جواب}$$

$$د) g^{-1} \circ f = g^{-1}(f(u)) = \{(3, 4), (7, 3), (9, 1)\} \Rightarrow \text{جواب}$$

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$$f = \{(2, 2), (2, 8), (4, 5)\} \text{ و } h = \{(1, 2), (3, 4), (2, 1), (5, 1)\}$$

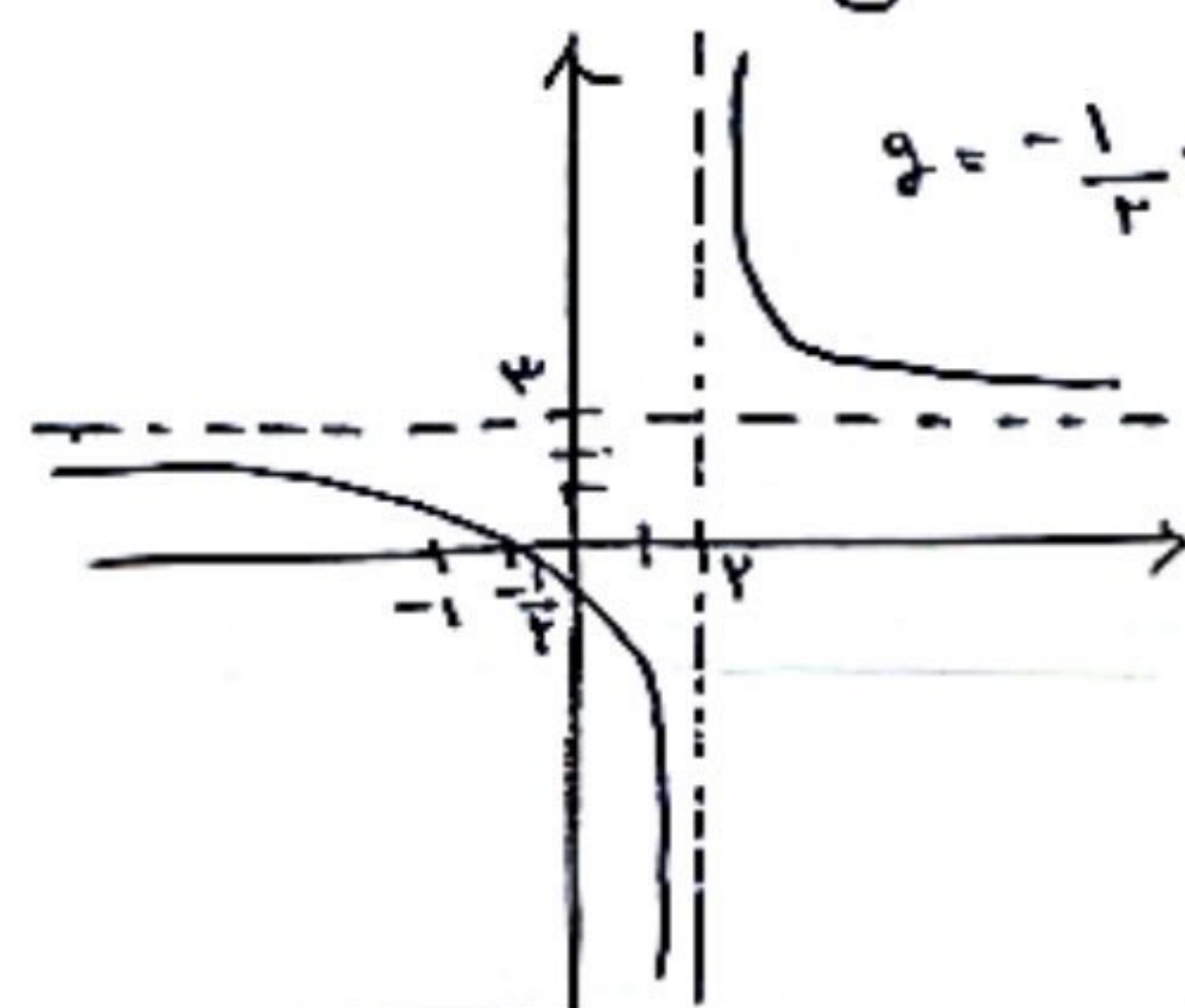
$$g = \{(2, 1), (2, 3), (4, 5)\} \Rightarrow g^{-1} = \{(1, 2), (3, 2), (5, 4)\}$$

$$\frac{h}{f(g^{-1}(u))} \Rightarrow \{(1, \frac{1}{2}), (3, \frac{1}{2})\} \Rightarrow \text{جواب}$$

$$\hookrightarrow \{(1, 2), (3, 8), (5, 5)\}$$

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$$y = \frac{3u+1}{u-2} \rightarrow \text{مشتق افقی است}$$



$$\frac{a}{c} = \frac{3x_0+1}{0-2}$$

$$y = -\frac{1}{2} = \frac{3x_0+1}{0-2}$$

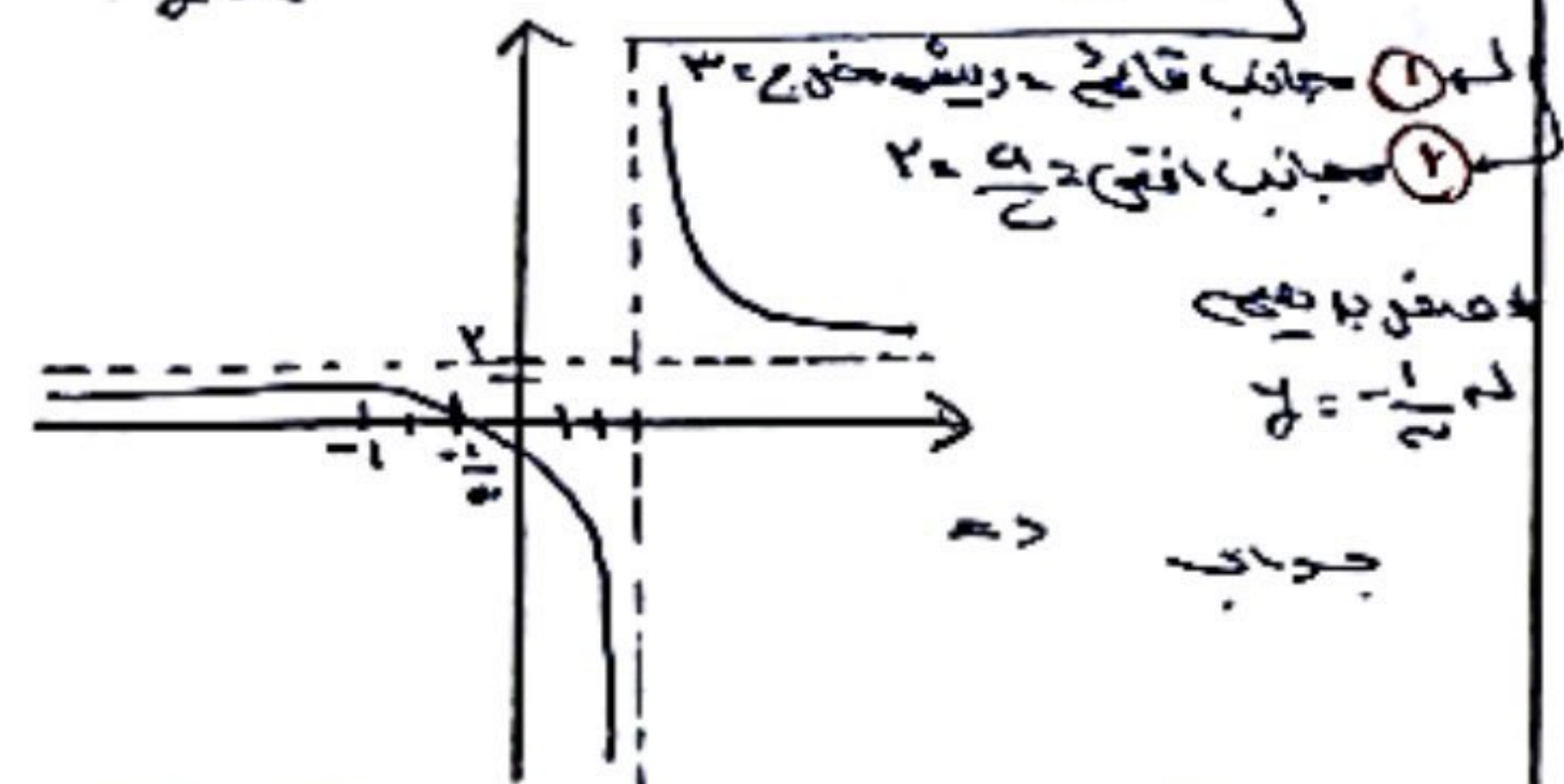
مشتق افقی است

یک به یک است
(مکروس پند)

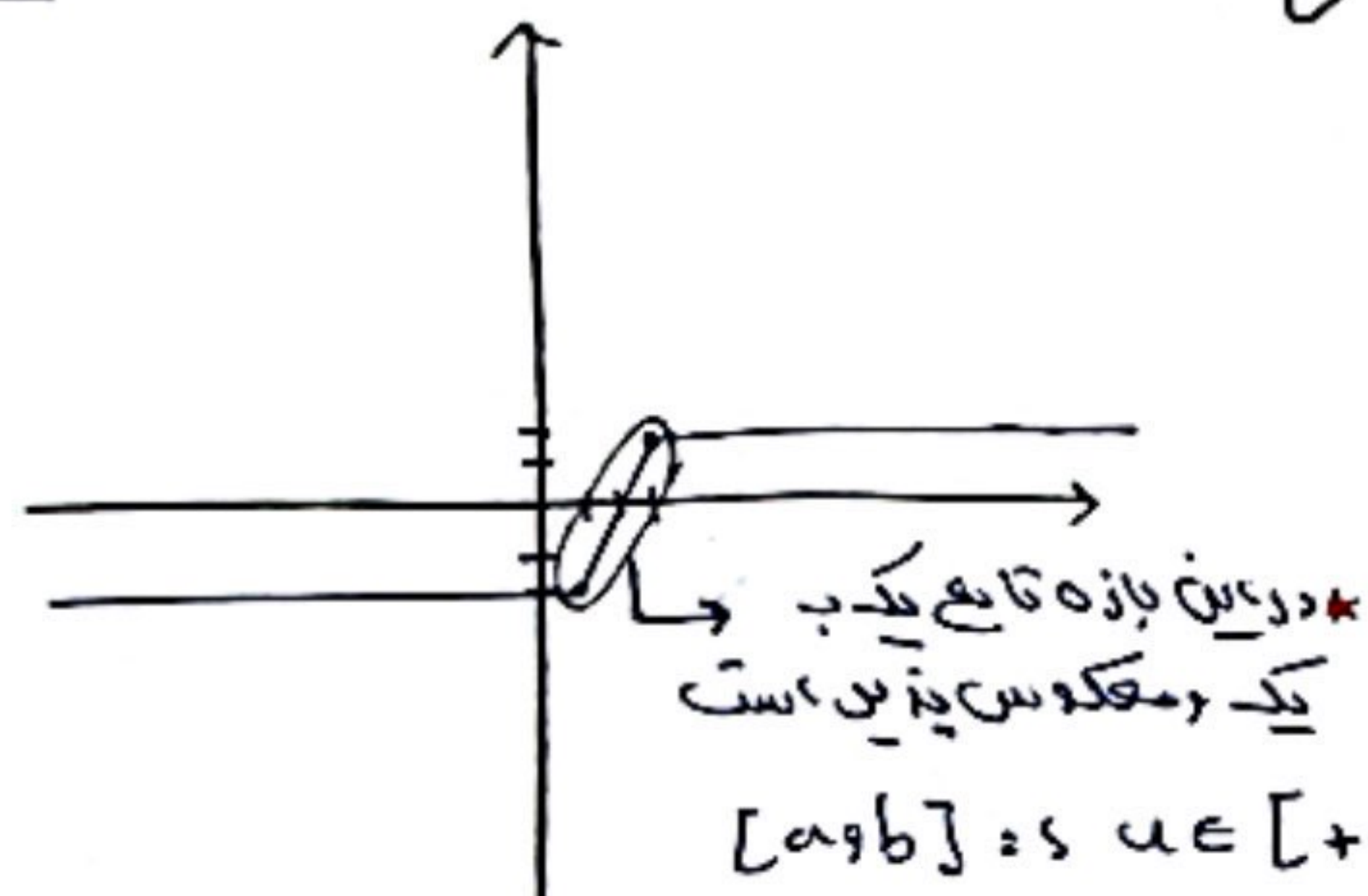
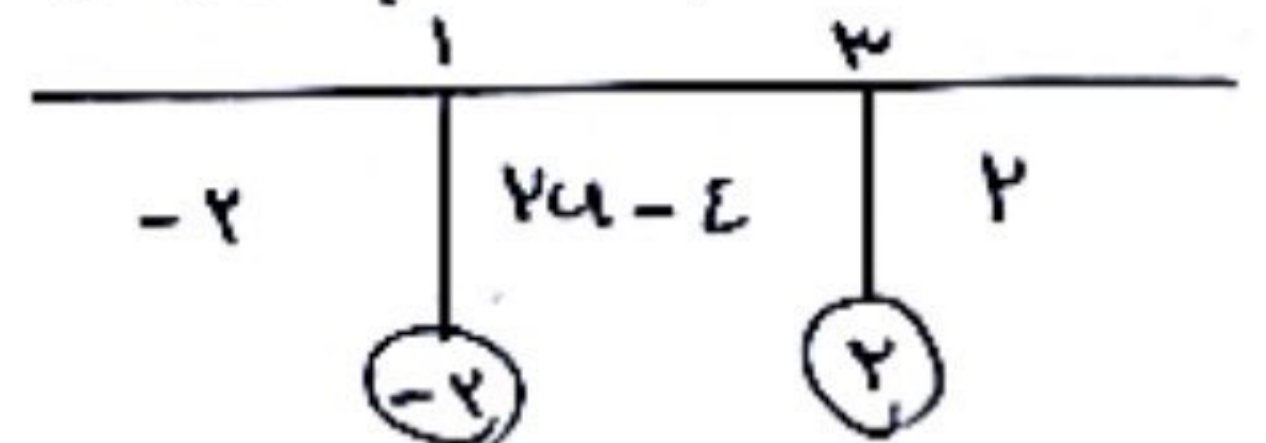
$$f^{-1} \Rightarrow y = \frac{3u+1}{u-2} \Rightarrow y(u-2) = 3u+1$$

$$\Rightarrow yu - 2y = 3u+1 \Rightarrow u(y-3) = 1+2y$$

$$\Rightarrow u = \frac{1+2y}{y-3} \Rightarrow f^{-1}(u) = \frac{1+2u}{u-3}$$



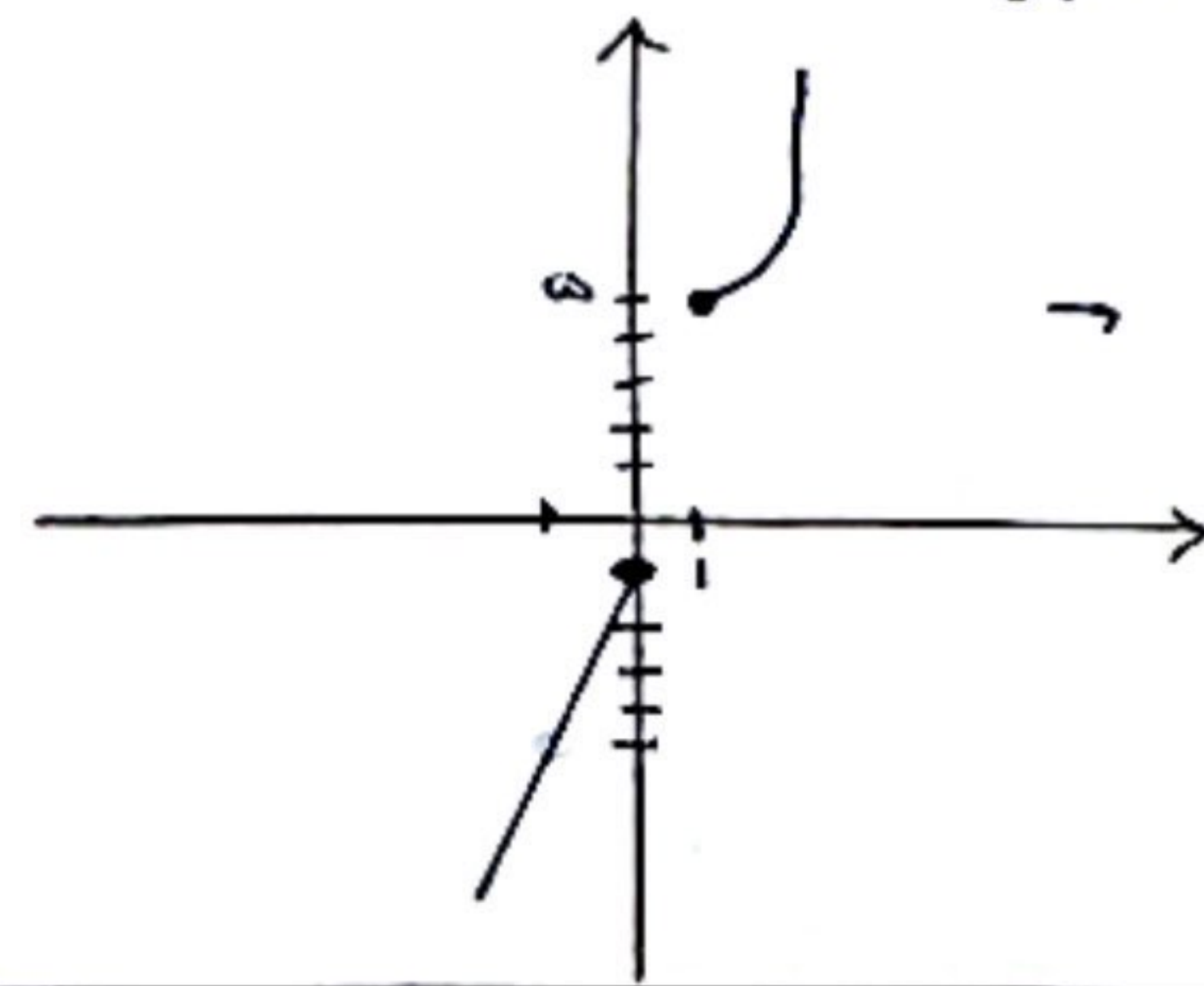
$$f(u) = |u-1| - |u-3|$$



$$0 \rightarrow y = 2u-2 \rightarrow u = y+1$$

$$y = u+2 \Rightarrow y = \frac{u}{2} + 2 \Rightarrow f^{-1}(u) = \frac{1}{2}u + 2$$

$$f(u) = \begin{cases} u^2+2 & u \geq 1 \\ 2u-1 & u < 1 \end{cases} \rightarrow \frac{u^2+2}{u^2-1} = \frac{u+1}{u-1}$$



یک به یک است
(واحد پند)

$$f^{-1} \Rightarrow y = u^2+2 \Rightarrow u = \sqrt{y-2} \Rightarrow y = u^2+2 \Rightarrow u = \sqrt{y-2}$$

$$f^{-1} \Rightarrow 2u-1 = y \Rightarrow 2y-1 = u \Rightarrow 2y = u+1 \Rightarrow y = \frac{u+1}{2}$$

$$f(u) = u^2 - \frac{(u+1)^2}{u+2} \Rightarrow f^{-1}(u) = \frac{au+b}{cu+d}$$

$$f(u) = u^2 - \frac{(u+1)^2}{u+2} \Rightarrow u^2(u+2) - (u^2+2u+1) = y(u+2) \Rightarrow y = \frac{u^3+2u^2-u^2-2u-1}{u+2}$$

$$\Rightarrow y = \frac{-u-1}{u+2} \Rightarrow y(u+2) = -u-1 \Rightarrow yu+2y = -u-1 \Rightarrow u(y+1) = -1-2y \Rightarrow u = \frac{-1-2y}{y+1}$$

$$\Rightarrow f^{-1}(u) = \frac{-1-2u}{u+2} \rightarrow \frac{ax-1-2u}{u+2} = \frac{au+b}{cu+d} \Rightarrow \frac{-2-4u}{u+2} = \frac{au+b}{u+d} \rightarrow \begin{cases} a=-2 \\ b=-2 \\ d=2 \end{cases}$$

$$f^{-1}(b) = f^{-1}(-2) = \frac{-1-2(-2)}{-2+2} = \infty \rightarrow \text{جواب}$$

$$f(u) = \frac{u}{u^2+1} \rightarrow y = \frac{u}{u^2+1} \rightarrow yu^2 - u + y = 0 \rightarrow \Delta = 1-4y^2$$

$$\Rightarrow u = \frac{1 \pm \sqrt{1-4y^2}}{2y} \rightarrow y \neq 0 \rightarrow y^{-1} = \frac{1 \pm \sqrt{1-4u^2}}{2u} \Rightarrow$$

پس ابتدا قابل قبول است

$$u \neq 1 \rightarrow f^{-1}(u) = \frac{1 + \sqrt{1-4u^2}}{2u} ; u \in [-\frac{1}{2}, \frac{1}{2}] \rightarrow \text{جواب}$$