

$$f(u) = \begin{cases} u^2 - 2 & ; u > a \\ 12u - 20 & ; u \leq a \end{cases}$$

$$u^2 - 2 = 12u - 20 \rightarrow u^2 - 12u + 18 = 0 \Rightarrow (u-2)^2 (u+2) \geq 0 \Rightarrow \boxed{a \geq -2}$$

که جواب

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$$f^{-1}(2) = 2 \rightarrow f(a) = b \Leftrightarrow f^{-1}(b) = a \Rightarrow f(2) = 2$$

$$f(u) = 3u + k \Rightarrow f(2) = 2 \Rightarrow f(2) = 3 \times 2 + k \Rightarrow 2 = 6 + k \Rightarrow \boxed{k = -4}$$

$$\textcircled{*} f(u) = 3u - 10$$

$$\text{الف) } f(u) = 3 \times 11 - 10 = 23 \rightarrow \text{جواب}$$

$$\text{ب) } f(f(u)) = 3(3u - 10) - 10 = 9u - 40 = 10 \Rightarrow \boxed{9u = 50} \rightarrow \text{جواب}$$

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$$f(u) = \frac{au}{u-1} \Rightarrow y = \frac{au}{u-1} \xrightarrow{\text{ضرب در } u-1} yu - y = au \Rightarrow yu - au = y \Rightarrow u(y-a) = y$$

$$\Rightarrow u = \frac{y}{y-a} \Rightarrow f^{-1}(u) = \frac{u}{u-a}$$

$$A(2a, a) \Rightarrow a = \frac{2a}{2a-a} \Rightarrow a = \frac{2a}{a} \Rightarrow \boxed{a=2}$$

که جواب

* نکته A روی این تابع پس مشخص
آنی باید در تابع صدق کند

۳

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$$g = \{(2, 4), (1, 2), (9, 5)\} \Rightarrow g^{-1} = \{(4, 2), (2, 1), (5, 9)\} \text{ و } f = \{(3, 5), (2, 4), (1, 2), (9, 5)\} \Rightarrow f^{-1} = \{(5, 3), (4, 2), (2, 1), (5, 9)\}$$

$$\text{الف) } f \circ f^{-1} = f(f^{-1}(u)) = \{(5, 5), (4, 4), (2, 2), (5, 9)\} \Rightarrow \text{جواب}$$

$$\text{ب) } f^{-1} \circ f = f^{-1}(f(u)) = \{(3, 3), (2, 2), (1, 1), (9, 9)\} \Rightarrow \text{جواب}$$

$$\text{ج) } f \circ g^{-1} = f(g^{-1}(u)) = \{(2, 5), (5, 4)\} \Rightarrow \text{جواب}$$

$$\text{د) } g^{-1} \circ f = g^{-1}(f(u)) = \{(3, 4), (4, 3), (9, 8)\} \Rightarrow \text{جواب}$$

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$$f = \{(2, 4), (4, 8), (4, 5)\} \text{ و } h = \{(1, 2), (3, 4), (2, 1), (5, 1)\}$$

$$g = \{(2, 1), (2, 3), (4, 5)\} \Rightarrow g^{-1} = \{(1, 2), (3, 2), (5, 4)\}$$

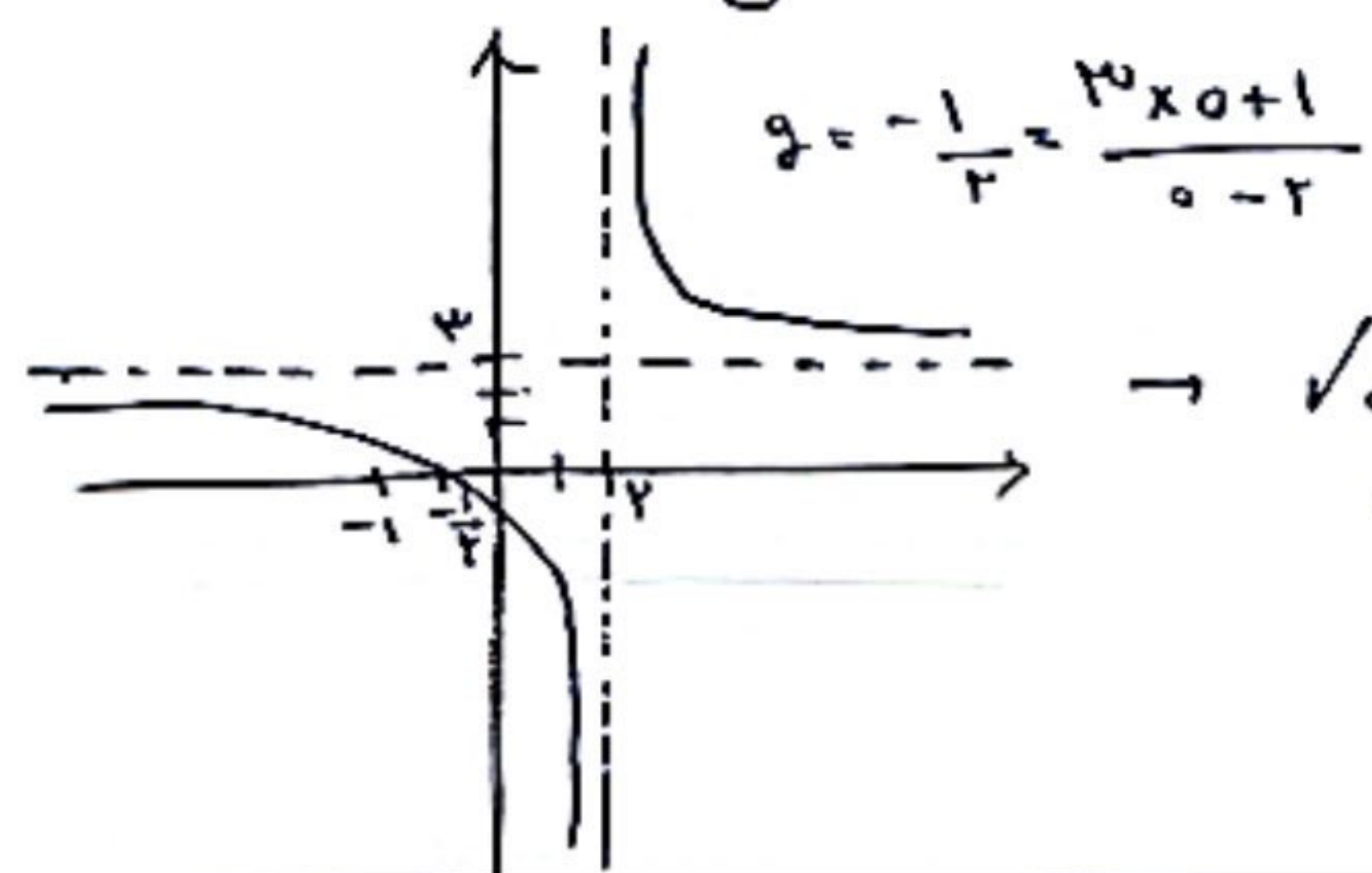
$$\frac{h}{f(g^{-1}(u))} \Rightarrow \{(1, \frac{1}{2}), (3, \frac{1}{2})\} \Rightarrow \text{جواب}$$

$$\hookrightarrow \{(1, 2), (3, 8), (5, 4)\}$$

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$$y = \frac{3u+1}{u-2} \rightarrow \text{مشتق افقی است}$$



$$\frac{a}{c} = \frac{3x_0+1}{0-2}$$

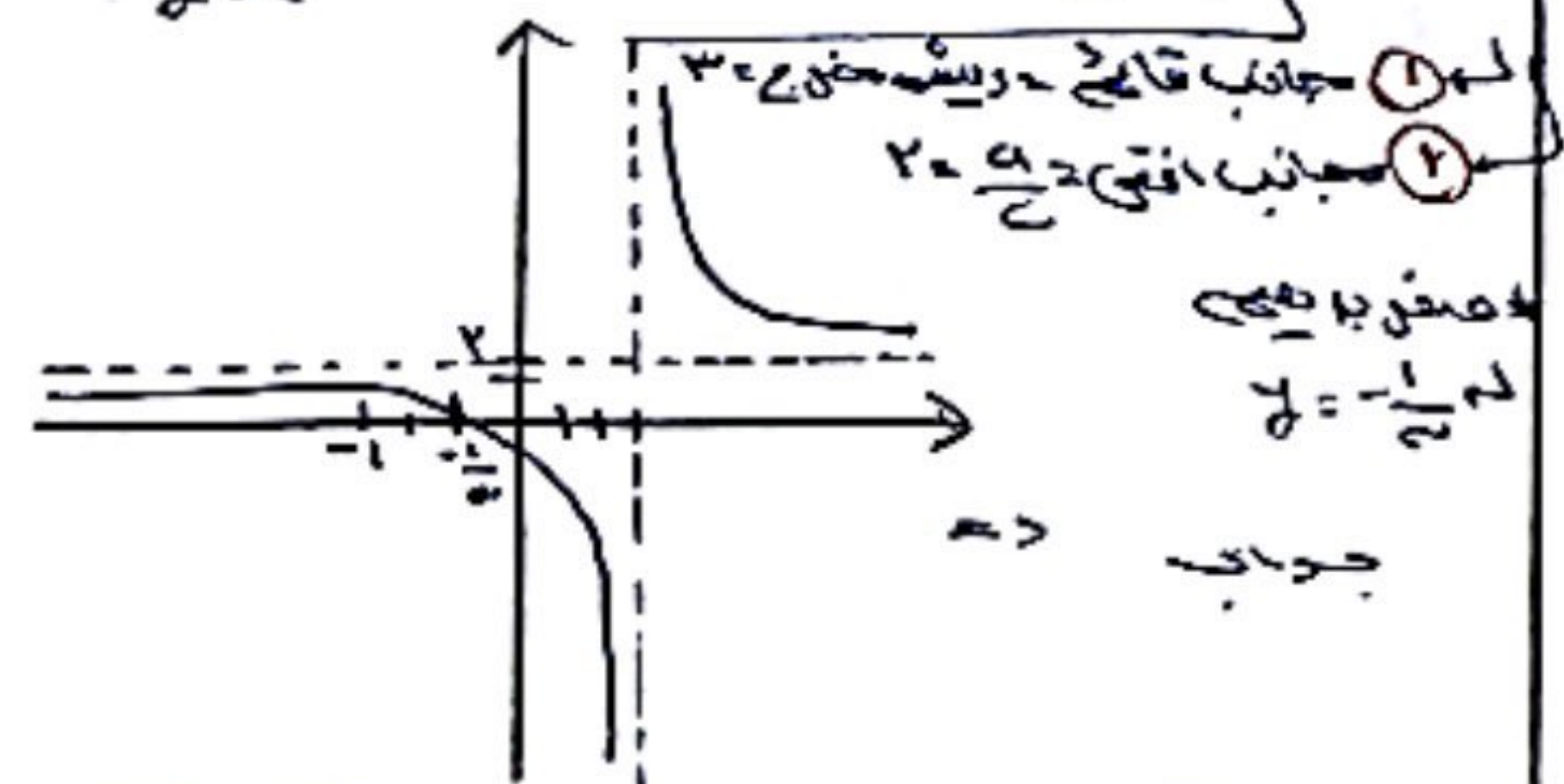
مشتق افقی است

یک به یک است
(مکروس پدید)

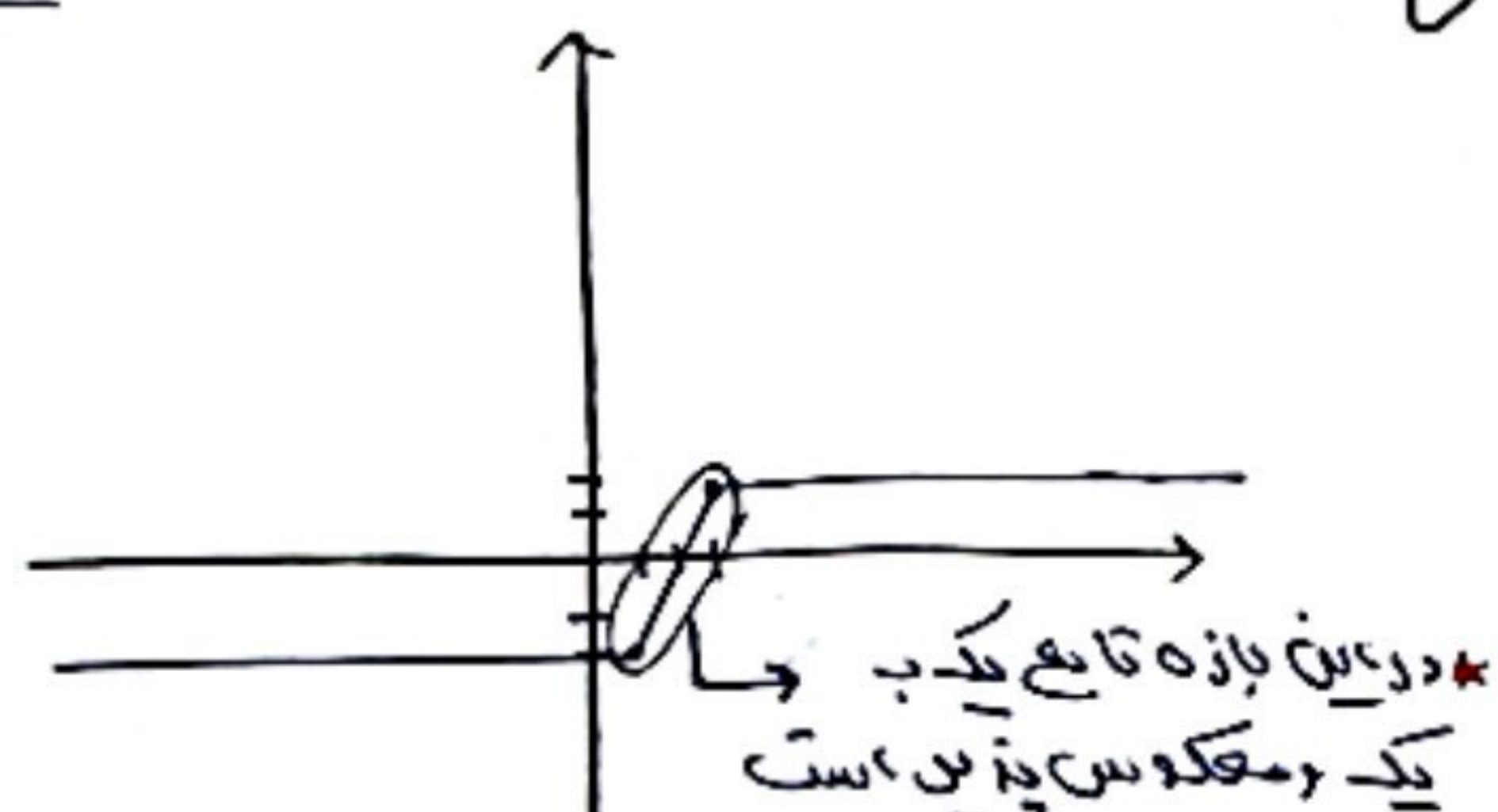
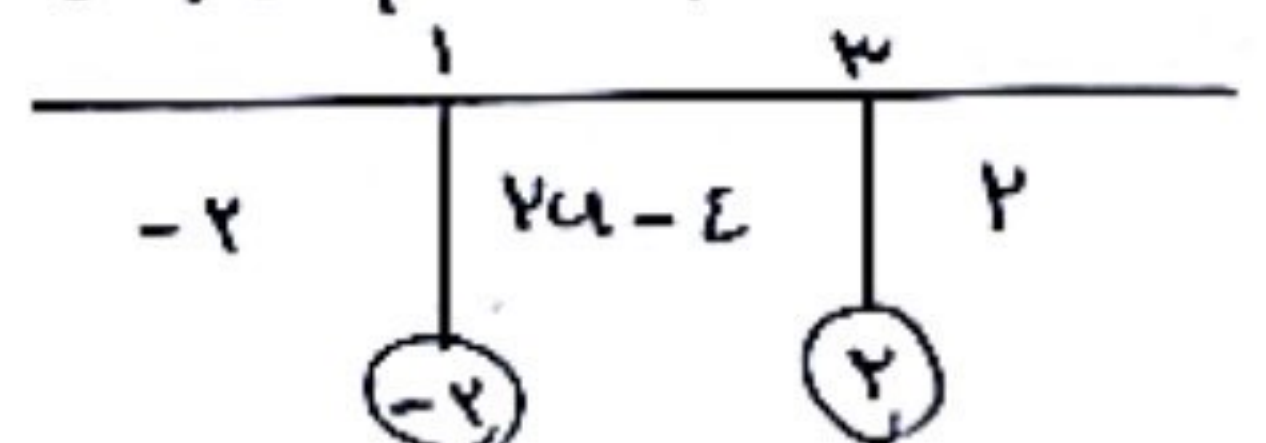
$$f^{-1} \Rightarrow y = \frac{3u+1}{u-2} \Rightarrow y(u-2) = 3u+1$$

$$\Rightarrow yu - 2y = 3u+1 \Rightarrow u(y-3) = 1+2y$$

$$\Rightarrow u = \frac{1+2y}{y-3} \Rightarrow f^{-1}(u) = \frac{1+2y}{y-3}$$



$$f(u) = |u-1| - |u-3|$$



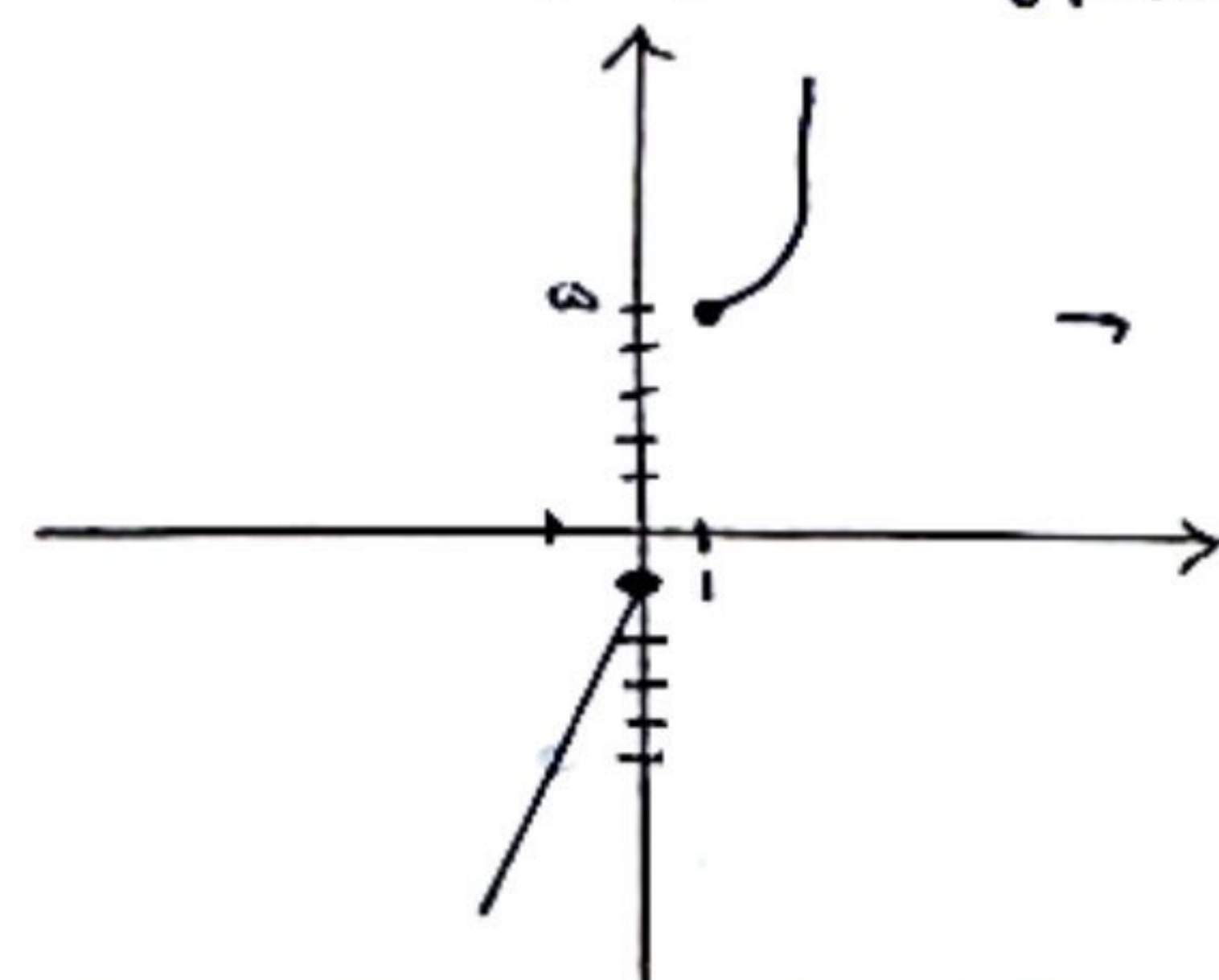
$$[a, b] : u \in [1, 3]$$

$$0 \rightarrow y = 2u-2 \xrightarrow{f^{-1}} u = \frac{y+2}{2} \Rightarrow$$

$$2y = u+2 \Rightarrow y = \frac{u}{2} + 2 \Rightarrow f^{-1}(u) = \frac{1}{2}u + 2$$

جواب

$$f(u) = \begin{cases} u^2+2 & u \geq 1 \\ 2u-1 & u < 1 \end{cases} \rightarrow \frac{u^2+2}{2} \rightarrow \frac{u^2+2}{2}$$



یک به یک است
(وارون پذیر است)

$$f^{-1} \Rightarrow y = u^2+2 \Rightarrow u = \sqrt{y-2} \Rightarrow y = u^2+2 \Rightarrow u = \sqrt{y-2}$$

$$f^{-1} \Rightarrow 2u-1 = y \Rightarrow 2y-1 = u \Rightarrow 2y = u+1 \Rightarrow y = \frac{u}{2} + \frac{1}{2}$$

$$f^{-1}(u) = \begin{cases} \sqrt{u-2} & u \geq 3 \\ \frac{1}{2}u + \frac{1}{2} & u < 3 \end{cases}$$

$$f(u) = u^2 - \frac{(u+1)^2}{u+3} \Rightarrow f^{-1}(u) = \frac{au+b}{cu+d}$$

$$f(u) = u^2 - \frac{(u+1)^2}{u+3} \Rightarrow u^2(u+3) - (u^2+2u+1) = y(u+3) \Rightarrow y = \frac{u^3+3u^2-u^2-2u-1}{u+3}$$

$$\Rightarrow y = \frac{-2u-1}{u+3} \Rightarrow yu+3y = -2u-1 \Rightarrow yu+2u = -1-3y \Rightarrow u(y+2) = -1-3y \Rightarrow u = \frac{-1-3y}{y+2}$$

$$\Rightarrow f^{-1}(u) = \frac{-1-3u}{u+3} \xrightarrow{f^{-1}} \frac{2x-1-3u}{2x} = \frac{au+b}{cu+d} \Rightarrow \frac{-2-4u}{2u+6} = \frac{au+b}{cu+d}$$

$$f^{-1}(b) = f^{-1}(-2) = \frac{-2-4(-2)}{-2+6} = \frac{6}{4} = \frac{3}{2} \rightarrow \text{جواب}$$

$$f(u) = \frac{u}{u^2+1} \rightarrow y = \frac{u}{u^2+1} \rightarrow yu^2 - u + y = 0 \rightarrow \Delta = 1-4y^2$$

$$\Rightarrow u = \frac{1 \pm \sqrt{1-4y^2}}{2y} \rightarrow y \neq 0 \rightarrow y^{-1} = \frac{1 \pm \sqrt{1-4u^2}}{2u} \Rightarrow$$

$$u \neq 1 \rightarrow f^{-1}(u) = \frac{1 + \sqrt{1-4u^2}}{2u} ; u \in [-\frac{1}{2}, \frac{1}{2}] \rightarrow \text{جواب}$$