

$$f(x) = \begin{cases} x^2 - \varepsilon & x > a \\ 12x - 20 & x \leq a \end{cases}$$

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$$x^2 - \varepsilon = 12x - 20$$

$$\Rightarrow x^2 - 12x + 14 = 0 \Rightarrow (x-2)(x+8) = 0$$

$$x = 2$$

$$x = -8 \text{ و } 8$$

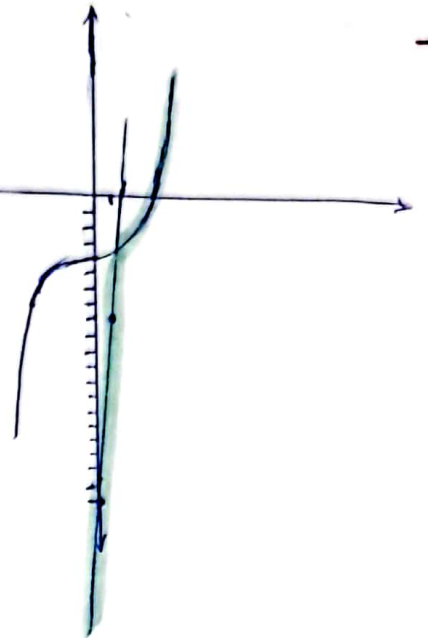
$$\begin{array}{r|l} x^2 - 12x + 14 & x - 2 \\ -x^2 + 12x^2 & \\ \hline 2x^2 - 12x + 14 & \end{array}$$

$$2x^2 - 12x + 14 \rightarrow (x-2)(x+8)$$

$$-2x^2 + 8x$$

$$\begin{array}{r} -10x + 14 \\ +10x - 14 \\ \hline 0 \end{array}$$

$$\Rightarrow \boxed{a \leq 2}$$



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$$f(x) = 3x + k \quad f^{-1}(2) = 7$$

$$x = 3y + k \Rightarrow 3y = x - k \Rightarrow y = \frac{1}{3}x - \frac{k}{3} \quad \begin{matrix} x=2 \\ y=7 \end{matrix} \rightarrow 7 = \frac{2}{3} - \frac{k}{3}$$

$$f(x) = 3x - 1$$

$$\Rightarrow \frac{k}{3} = -\frac{1}{3} \Rightarrow k = -1$$

$$\text{الف) } f(2) = 3 \cdot 2 - 1 = 6 - 1 = 5 \quad \text{ب) } f(f(x)) = 3(3x - 1) - 1 = 9x - 3 - 1 = 9x - 4$$

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$$f(x) = \frac{ax}{x-1}, A(2a, a) \rightarrow f(x) \text{ در } x=2a$$

$$f^{-1}(x): x = \frac{ay}{y-1} \Rightarrow xy - x = ay \Rightarrow x = (x-a)y \Rightarrow y = \frac{x}{x-a}$$

$$\xrightarrow{x=2a} a = \frac{2a}{2a-a} = 2$$

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$$f = \{(3, 5), (4, 6), (5, 7), (6, 8)\} \quad g = \{(3, 2), (4, 3), (5, 4)\}$$

$$f^{-1} = \{(5, 3), (6, 4), (7, 5), (8, 6)\} \quad g^{-1} = \{(2, 3), (3, 4), (4, 5)\}$$

$$\text{الف) } f \circ f^{-1} = \{(5, 5), (6, 6), (7, 7), (8, 8)\}$$

$$b) f^{-1} \circ f = \{(3,3) (4,4) (5,5) (6,6)\}$$

$$c) f \circ g^{-1} = \{(1,4) (4,1)\}$$

$$d) g^{-1} \circ f = \{(3,4) (4,3) (6,1)\}$$

$$f = \{(1,4) (3,1) (4,0)\} \quad g = \{(1,1) (3,3) (4,4)\} \quad h = \{(1,2) (3,4) (4,1) (6,1)\}$$

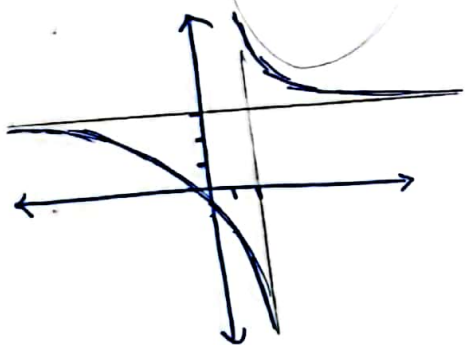
$$g^{-1} = \{(1,3) (3,1) (4,4)\}$$

$$f \circ g^{-1} = \{(1,1) (3,3) (4,0)\}$$

$$\frac{h}{f \circ g^{-1}} = \{(1, \frac{1}{4}) (3, \frac{1}{1}) (\cancel{4, \frac{1}{0}})\} = \{(1, \frac{1}{4}) (3, \frac{1}{1})\}$$

$$y = \frac{3x+1}{x-2}$$

پارتنه به سهولت
یابید یا نیست



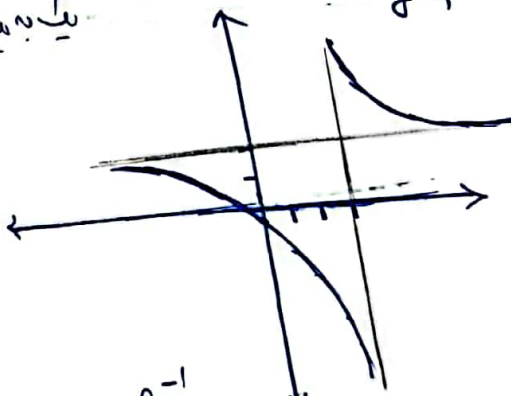
$f(x)$

$$x = \frac{3y+1}{y-2}$$

$$\Rightarrow xy - 2x = 3y + 1$$

$$(x-3)y = 2x+1$$

$$y = \frac{2x+1}{x-3}$$



$f^{-1}(x)$

$$f(x) = |x-1| - |x-3|$$

	1	3	
$-x+1+x-3$	$x-1+x-3$	$x-1-x+3$	
$= -2$	$= 2x-4$	$= 2$	

$$y = 2x-4$$

$$x = 2y-4 \Rightarrow 2y = x+4$$

$$\Rightarrow y = \frac{1}{2}x + 2$$

$$\Rightarrow f^{-1}(x) = \frac{1}{2}x + 2$$

$$D_f = R_{f^{-1}} \Rightarrow R_{f^{-1}} = [1, 3]$$

$$1 \leq \frac{1}{2}x + 2 \leq 3 \Rightarrow -2 \leq x \leq 2$$

$$\Rightarrow D_{f^{-1}} = [-2, 2]$$

← تابع در بازه $[1, 3]$ به صورت
خطی و وارون پذیر است.

$$f(x) = \begin{cases} x^3 + \varepsilon & x > 1 \\ x - 1 & x \leq 0 \end{cases}$$

$$x = y^3 + \varepsilon \quad y > 1$$

$$\Rightarrow y = \sqrt[3]{x - \varepsilon}$$

$$\sqrt[3]{x - \varepsilon} > 1 \Rightarrow x - \varepsilon > 1$$

$$\Rightarrow x > 1 + \varepsilon$$

$$x = \varepsilon y - 1 \quad y \leq 0$$

$$\Rightarrow \varepsilon y = x + 1$$

$$\Rightarrow y = \frac{x}{\varepsilon} + \frac{1}{\varepsilon}$$

$$y \leq 0 \Rightarrow \varepsilon y \leq 0 \Rightarrow x + 1 \leq 0$$

$$\Rightarrow x \leq -1$$

$$f^{-1}(x) = \begin{cases} \sqrt[3]{x - \varepsilon} & x > 1 + \varepsilon \\ \frac{x}{\varepsilon} + \frac{1}{\varepsilon} & x \leq -1 \end{cases}$$

$$f(x) = x^2 - \frac{(x+1)^2}{x+3} \quad f'(x) = \frac{ax+b}{cx+d} \quad f^{-1}(b) = ?$$

$$\hookrightarrow \frac{x^2 + 3x^2 - (x^2 + 3x + 3x^2 + 1)}{x+3} = \frac{-3x-1}{x+3}$$

$$f^{-1}(x): \quad x = \frac{-3y-1}{y+3} \Rightarrow xy + 3x = -3y - 1$$

$$(x+3)y = -3x-1 \Rightarrow y = \frac{-3x-1}{x+3}$$

$$= \frac{-4x-2}{x+3} \quad \sim \quad a = -4$$

$$b = -2$$

$$d = 3$$

$$f^{-1}(-2) = \frac{4-1}{-2+3} = 3$$

$$f(x) = \frac{x}{x^2+1} \quad \sim \quad f^{-1}(x): \quad x = \frac{y}{y^2+1} \Rightarrow xy^2 + x = y$$

$$\Rightarrow xy^2 - y + x = 0 \quad y = \frac{1 \pm \sqrt{1-4x^2}}{2x} \quad \text{for } x, y > 0 \rightarrow y = \frac{1 + \sqrt{1-4x^2}}{2x}$$