

1. $x^r - \varepsilon = f(n) \rightarrow R = (a^r - \varepsilon, +\infty)$, $1 \leq n < r$, $f(n) \rightarrow R = (-\infty, 1 \leq a - r_0)$
 $1 \leq a - r_0 \leq a^r - \varepsilon \rightarrow a^r - 1 \leq 1 \leq a^r \rightarrow (a-1)(a-1)(a+\varepsilon) \geq 0 \rightarrow \begin{array}{c|c} -\varepsilon & * \\ -1 & + \\ + & + \end{array}$

2. $f(n) = r_n + k$ $f^{-1}(r) = \varepsilon \Rightarrow f(\varepsilon) = r \Rightarrow f(\varepsilon) = 1 + k = r \Rightarrow k = r - 1 \Rightarrow f(n) = r_n - 1$

و) $f(r) = r_n - 1 = 1$

و) $f(f(n)) = f(r_n - 1) = r_n - r_0 - 1 = r_n - \varepsilon$

3. $f(n) = \frac{a^n}{n-1}$ $A(r, a)$

$\Rightarrow A(a) = r_a \rightarrow f(n) = \frac{a^r}{a-1} = r_a \neq \frac{a+1}{a-1}$, $a^r = r_a^r - r_a \Rightarrow a^r - r_a = 0 \Rightarrow \left\{ \begin{array}{l} a = 0 \\ a = r \end{array} \right\} \neq a = r$

4. و) $f \circ f^{-1} = \{(a, a)(r, r)(q, q)\}$

و) $f^{-1} \circ f = \{(r, r)(q, q)(\varepsilon, \varepsilon)(a, a)\}$

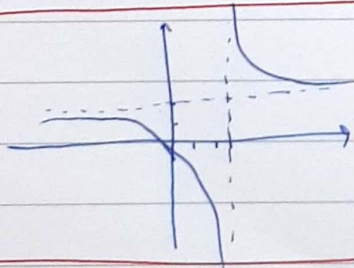
و) $f \circ g^{-1} = \{(r, a)(a, q)\}$

و) $g^{-1} \circ f = \{(r, r)(r, a)(a, a)\}$

5. $\frac{h}{f \circ g^{-1}} = g^{-1}(n) \{ (1, r)(r, \varepsilon)(a, q) \} \rightarrow f \circ g^{-1}(n) = \{ (1, \varepsilon)(r, a)(a, 0) \}$

$\frac{h}{f \circ g^{-1}} = \frac{r}{\varepsilon} = \frac{1}{r}, \frac{\varepsilon}{r}, \frac{1}{0} \rightarrow \frac{h}{f \circ g^{-1}(n)} = \{ (r, \frac{1}{r})(1, \frac{1}{r}) \}$

6. $y = \frac{r_{n+1}}{n-r}$

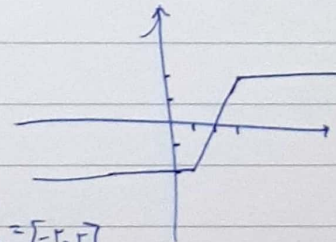


$\Rightarrow \frac{h}{f \circ g^{-1}} = y = \frac{r_{n+1}}{n-r} \rightarrow f(n) = \frac{r_{n+1}}{n-r}$
 $\Rightarrow f(n) = \frac{r_{n+1}}{n-r}$

7. $y = |n-1| - |n-3| \Rightarrow$

$\Rightarrow [a, b] = [1, 3] \rightarrow a=1, b=3$

$\hookrightarrow y = r_n \pm \varepsilon, n \in [1, 3] \rightarrow y = \frac{n+\varepsilon}{r} \Rightarrow R_y = D_{f^{-1}} = [1, 3]$



$$8. f(n) = \begin{cases} n^r + \varepsilon & ; n \geq 1 \\ \varepsilon n - 1 & ; n \leq 0 \end{cases}$$

$$Df^{-1}([a, +\infty))$$

$$f^{-1}(n) \begin{cases} y^r + \varepsilon = n \rightarrow n - \varepsilon = y^r \Rightarrow y = \sqrt[r]{n - \varepsilon} \\ \varepsilon y - 1 = n \rightarrow \varepsilon y = n + 1 \Rightarrow y = \frac{n + 1}{\varepsilon} \end{cases}$$

$$Df^{-1} = [-\infty, -1]$$

$$9. f(n) = \frac{n^r(n+1)^p}{n+p} \rightarrow f^{-1}(n) = \frac{a_n + b}{r_n + d}$$

$$\frac{n^r(n+p)}{n+p} = \frac{n^r + 1 + r n^r + p n}{n+p} = \frac{-r n - 1}{n+p} \times \frac{p}{p} = \frac{-r n - p}{r n + p} \Rightarrow a = -r, b = -p, d = p$$

$$\Rightarrow f^{-1}(b) = \frac{p - 1}{-r + p} = a$$

$$10. f(n) = \frac{n}{n^r + 1} \Rightarrow n = \frac{y}{y^r + 1} \Rightarrow n y^r + n = y \Rightarrow n y^r + y + n = 0 \Rightarrow y = \frac{-1 \pm \sqrt{1 - 4 n^r}}{2 n}$$

$$\begin{cases} \frac{-1 - \sqrt{1 - 4 n^r}}{2 n} \\ \frac{-1 + \sqrt{1 - 4 n^r}}{2 n} \end{cases}$$

$$1 - 4 n^r > 0 \rightarrow n^r < \frac{1}{4} \rightarrow -\frac{1}{r} < n \leq \frac{1}{r} \Rightarrow n \in \left[-\frac{1}{r}, \frac{1}{r}\right], n \neq 0$$