یازدهم ریاضی C   فاطمه توکلی - ۱۱

۱۹,۷۵

Date   No

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1- $x^3 - 4 = f(x) \to R = (a^3-4, +\infty)$, $12x-20 = f(x) \to R = (-\infty, 12a-20)$

$12a-20 \geq a^3-4 \to a^3-12+16 \geq 0 \to (a-2)(a-2)(a+4) \geq 0 \to$

	-4		2	
$-$	0	$+$	0	$+$

$a \in [-4, +\infty)$

2- $f(x) = 3x+k$ $\quad f^{-1}(2) = 4 \Rightarrow f(4) = 2 \Rightarrow f(4) = 12+k = 2 \Rightarrow k = -10 \Rightarrow f(x) = 3x-10$

الف) $f(7) = 3\times 7 - 10 = 11$

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ب) $f(f(x)) = f(3x-10) = 9x-30-10 = 9x-40$

3- $f(x) = \dfrac{ax}{x-1}$ $\quad A(2a, a)$

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$\Rightarrow f(a) = 2a \to f(a) = \dfrac{a^2}{a-1} = 2a \xrightarrow{a \neq 1} a^2 = 2a^2 - 2a \Rightarrow a^2 - 2a = 0 \Rightarrow \begin{cases} a = 0 & \text{غیرقابل قبول} \\ a = 2 \end{cases} \Rightarrow a = 2$

4- الف) $f \circ f^{-1} = \{(5,5)(7,7)(2,2)(6,6)\}$

ب) $f^{-1} \circ f = \{(3,3)(7,7)(4,4)(9,9)\}$

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ج) $f \circ g^{-1} = \{(3,5)(5,6)\}$

د) $g^{-1} \circ f = \{(7,3)(3,9)(9,8)\}$

5- $\dfrac{h}{f \circ g^{-1}} \to g^{-1}(x) = \{(1,2)(3,4)(5,6)\} \to f \circ g^{-1}(x) = \{(1,4)(3,8)(5,0)\}$

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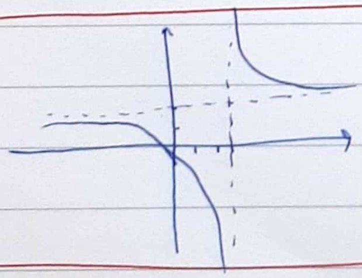
$\to \dfrac{h}{f \circ g^{-1}} = \dfrac{2}{4} = \dfrac{1}{2}, \dfrac{4}{8}, \dfrac{1}{0}$ تعریف نشده $\Rightarrow \dfrac{h(x)}{f \circ g^{-1}(x)} = \{(3, \tfrac{1}{2})(1, \tfrac{1}{2})\}$

6- $y = \dfrac{3x+1}{x-2}$



$\Rightarrow$ تابع یک به یک است $\Rightarrow y = \dfrac{2x+1}{x-2} \to f(x) = \dfrac{-2x-1}{x-3}$

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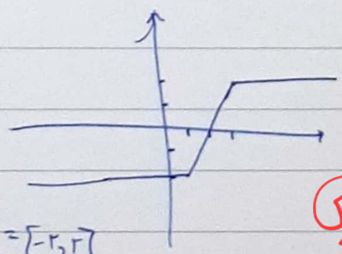
$\Rightarrow f(x) = \dfrac{2x+1}{x-3}$

7- $y = |x-1| - |x-3| \Rightarrow$

	1		3	
$-x+1+x-3$		$2x-4$		$x-1-x+3$
-2	-2		2	2



$\Rightarrow [a,b] = [1,3] \to a = 1, b = 3$

$y = 2x-4, x \in [1,3] \to y^{-1} = \dfrac{x+4}{2} \Rightarrow R_y = D_{y^{-1}} = [-2,2]$

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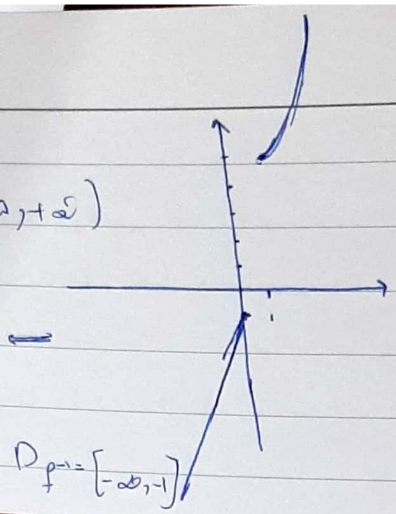
8 - $f(x) = \begin{cases} x^3+4 & ; x \geq 1 \\ 4x-1 & ; x \leq 0 \end{cases}$

$D_{f^{-1}} = [5, +\infty)$

$f^{-1}(x) \begin{cases} y^3+4 = x \to x-4 = y^3 \Rightarrow y = \sqrt[3]{x-4} \\ 4y-1 = x \to 4y = x+1 \Rightarrow y = \dfrac{x+1}{4} \searrow D_{f^{-1}} = (-\infty, -1] \end{cases}$



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9 - $f(x) = x^2 - \dfrac{(x+1)^3}{x+3}$ — $f^{-1}(x) = \dfrac{ax+b}{2x+d}$

$\dfrac{x^2(x+3)}{x+3} - \dfrac{x^3+1+3x^2+3x}{x+3} = \dfrac{-3x-1}{x+3} \times \dfrac{2}{2} = \dfrac{-6x-2}{2x+6} \Rightarrow a=-6, b=-2, d=6$

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$\Rightarrow f^{-1}(b) = \dfrac{6-1}{-2+3} = 5$

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10 - $f(x) = \dfrac{x}{x^2+1} \Rightarrow x = \dfrac{y}{y^2+1} \Rightarrow xy^2 + x = y \Rightarrow xy^2 + y + x = 0 \Rightarrow y = \dfrac{-1 \pm \sqrt{1-4x^2}}{2x}$

$\begin{cases} \dfrac{-1-\sqrt{-4x^2+1}}{2x} \\ \dfrac{-1+\sqrt{-4x^2+1}}{2x} \end{cases}$

$1-4x^2 > 0 \to x^2 < \frac{1}{4} \to -\frac{1}{2} < x \leq \frac{1}{2} \Rightarrow x \in [-\frac{1}{2}, \frac{1}{2}], x \neq 0$

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