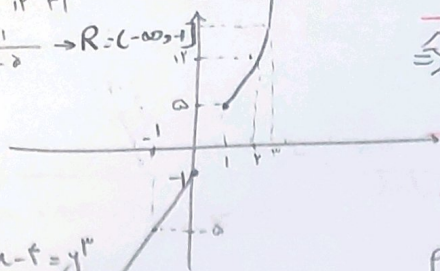


$$f(x) = \begin{cases} x^3 + 4 & x \geq 1 \\ 4x - 1 & x < 0 \end{cases} \quad \begin{matrix} \begin{matrix} 1 & 2 & 3 \\ \Delta & 12 & 31 \end{matrix} \rightarrow R = [\Delta, +\infty) \\ \begin{matrix} 0 & -1 \\ -1 & -2 \end{matrix} \rightarrow R = (-\infty, +1] \end{matrix}$$



تابع وارون است

$$y = x^3 + 4 \xrightarrow{\text{وارون}} x = y^3 + 4 \rightarrow x - 4 = y^3 \rightarrow y = \sqrt[3]{x-4}$$

$$f^{-1}(x) = \begin{cases} \sqrt[3]{x-4} & , x \geq 5 \\ \frac{x+1}{4} & , x < -1 \end{cases}$$

$$y = 4x - 1 \xrightarrow{\text{وارون}} x = \frac{y+1}{4} \rightarrow x+1 = \frac{y+1}{4} \rightarrow y = \frac{x+1}{4}$$

$$f(x) = x^3 - \frac{(x+1)^3}{x+3}$$

$$f^{-1}(x) = \frac{ax+b}{cx+d}$$

$$f^{-1}(b) = ? \quad (b < 0)$$

مسئله ۹

$$x^3 - \frac{(x+1)^3}{x+3} \xrightarrow{\text{وارون}} \frac{x^3(x+3) - (x+1)^3}{x+3} = \frac{x^4 + 3x^3 - (x^3 + 3x^2 + 3x + 1)}{x+3} = \frac{x^4 + 2x^3 - 3x^2 - 3x - 1}{x+3}$$

$$= \frac{-3x^2 - 1}{x+3} \xrightarrow{\text{وارون}} y = \frac{-3x^2 - 1}{x+3} \Rightarrow x = \frac{-3y - 1}{y+3} \rightarrow xy + 3x = -3y - 1 \rightarrow xy + 3y = -1 - 3x$$

$$\rightarrow y(x+3) = -1 - 3x \rightarrow y = \frac{-1-3x}{x+3} \xrightarrow{\text{وارون}} f^{-1}(x) = \frac{-4x-2}{2x+4} \rightarrow b = -2$$

$$f^{-1}(-2) = \frac{-4(-2)-2}{2(-2)+4} = \frac{8-2}{-4+4} = \frac{6}{0} = \Delta$$

$$f(x) = \frac{x}{x^2+1}$$

$$f^{-1}(x) = ? \rightarrow y = \frac{x}{x^2+1} \xrightarrow{\text{وارون}} x = \frac{y}{y^2+1}$$

چون $x \in [-1, 1]$ و $y \in [-1, 1]$ هم داریم

مسئله ۱۰

$$x = \frac{y}{y^2+1} \rightarrow xy^2 + y + x = 0 \rightarrow \Delta = 1 \pm \sqrt{1-4x^2} \Rightarrow f^{-1} = \frac{1 \pm \sqrt{1-4x^2}}{2x} \quad (x \in [-\frac{1}{2}, \frac{1}{2}])$$

$$R(f) = D(f^{-1}) \rightarrow R(f) = [-\frac{1}{2}, \frac{1}{2}]$$

$$(-\infty, -\frac{1}{2}) \cup (\frac{1}{2}, +\infty) \leftarrow \left(\frac{1}{2x+1} \right)$$