

زهرا جیوانی / مسیحیاریه II / یازدهم فصل

$$12a - 2 \leq a^3 - \varepsilon$$

$$a^3 - 12a + 14 \geq 0 \rightarrow (a-2)^2 (a+7) \geq 0$$

$$\begin{array}{c} \star \\ -\varepsilon \quad 2 \\ \hline - \quad + \quad + \end{array} \rightarrow [-\varepsilon, +\infty)$$

$$f(n) = 3n + k \rightarrow f^{-1}(n) \Rightarrow 3y + k = n$$

$$f^{-1}(k) = \varepsilon \quad 3y = n - k \rightarrow f^{-1}(n) = \frac{n - k}{3}$$

$$\frac{n - k}{3} = \varepsilon \rightarrow n - k = 15 \rightarrow k \leq -10$$

$$\textcircled{1} f(v) = 31 - 10 = 11 \quad \textcircled{2} f(f(n)) = 3(3n - 10) - 10 = 9n - 40$$

$$f^{-1}(n) \Rightarrow \frac{ay}{y-1} = n \rightarrow ny - n = ay \rightarrow (n-a)y = n$$

$$\rightarrow y = \frac{n}{n-a} = f^{-1}(n)$$

$$\rightarrow n \leq 2a \rightarrow \frac{2a}{a} = 2 = a$$

Arman

$$g^{-1}(m) = (1, 3)(4, 1)(2, 9)$$

$$f^{-1}(m) = \{(2, 3)(1, 4)(1, 9)(4, 9)\}$$

- 2

$$(1) f(f^{-1}(m)) \Rightarrow \begin{matrix} 2 \rightarrow 3 \rightarrow 2 & 1 \rightarrow 4 \rightarrow 1 \\ 4 \rightarrow 1 \rightarrow 4 & 9 \rightarrow 9 \rightarrow 9 \end{matrix} \Rightarrow \{(2, 2)(1, 1)(4, 4)(9, 9)\}$$

$$(2) f^{-1}(f(m)) \Rightarrow \begin{matrix} 3 \rightarrow 2 \rightarrow 3 & 4 \rightarrow 1 \rightarrow 4 \\ 1 \rightarrow 4 \rightarrow 1 & 9 \rightarrow 9 \rightarrow 9 \end{matrix} \Rightarrow \{(3, 3)(1, 1)(4, 4)(9, 9)\}$$

$$(3) f(g^{-1}(m)) \Rightarrow \begin{matrix} 1 \rightarrow 3 \rightarrow 2 \\ 4 \rightarrow 1 \rightarrow 4 \\ 9 \rightarrow 9 \rightarrow 9 \end{matrix} \Rightarrow \{(1, 2)(2, 4)\}$$

$$(4) g^{-1}(f(m)) \Rightarrow \begin{matrix} 3 \rightarrow 2 \rightarrow 1 \\ 1 \rightarrow 4 \rightarrow 1 \\ 9 \rightarrow 9 \rightarrow 9 \end{matrix} \Rightarrow \{(3, 1)(1, 4)(9, 9)\}$$

$$g^{-1}(m) = (1, 2)(3, 4)(2, 9)$$

- 3

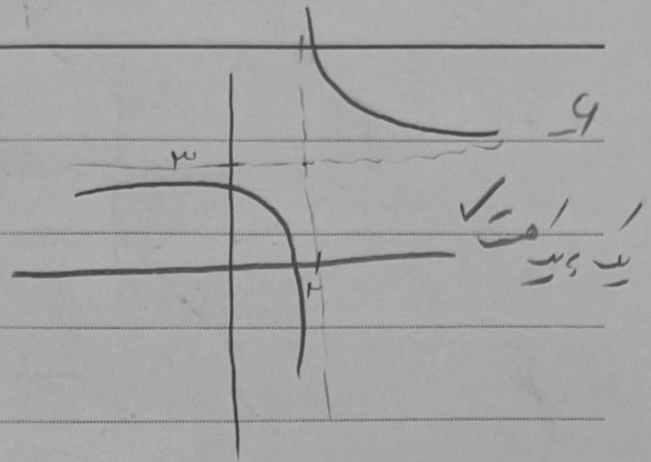
$$f(g^{-1}(m)) = \begin{matrix} 1 \rightarrow 2 \rightarrow 1 \\ 3 \rightarrow 4 \rightarrow 3 \\ 2 \rightarrow 9 \rightarrow 2 \end{matrix} \Rightarrow \{(1, 2)(3, 4)(2, 2)\}$$

$$\frac{h}{\log^{-1}} \Rightarrow \begin{matrix} 1, \frac{1}{2} \\ 3, \frac{1}{4} \\ 2, \frac{1}{9} \end{matrix} \Rightarrow \{(1, \frac{1}{2})(3, \frac{1}{4})\}$$

2 و 1/2 → مقلوب ×

Arman

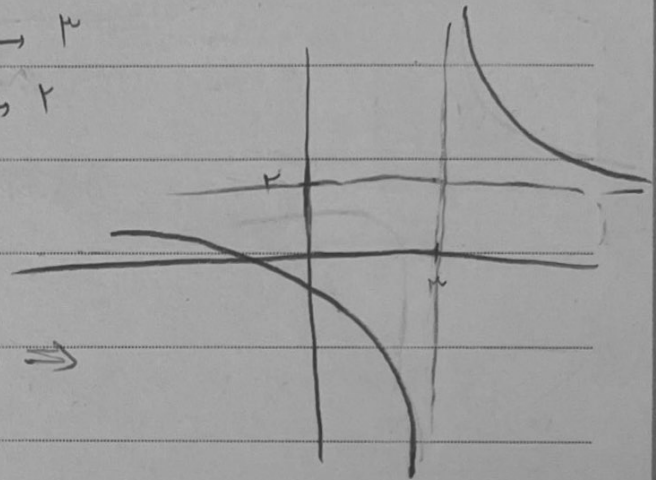
$$f(x) = \frac{r_n + 1}{x - r} \quad \begin{matrix} \rightarrow r = \text{نقطهٔ بلیز} \\ \rightarrow r = \frac{a}{c} \end{matrix}$$



$$x = \frac{ry + 1}{y - r}$$

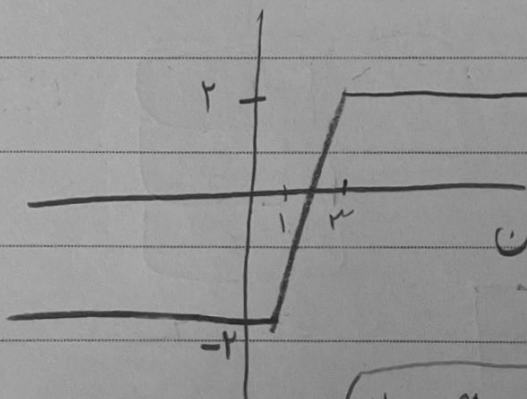
$$xy - rn = ry + 1 \rightarrow (x - r)y = rn + 1$$

$$y = f^{-1}(x) = \frac{rn + 1}{x - r} \quad \begin{matrix} \rightarrow r \\ \rightarrow r \end{matrix}$$



$f^{-1}(x)$ و $f(x)$ متقابلند

$$y = |x - 1| - |x - r| \quad \begin{matrix} 1 & r \\ -r & rn - \varepsilon & r \end{matrix}$$



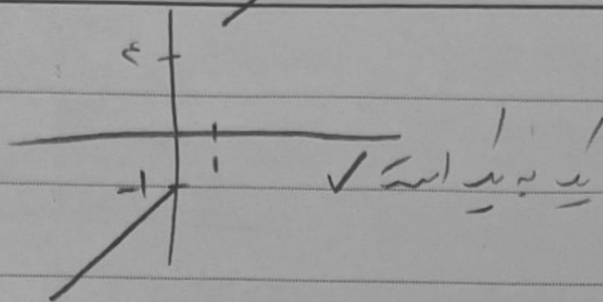
$$[a, b] = [1, r]$$

$$\text{پاره‌ها} \Rightarrow ry - \varepsilon = x \rightarrow y = \frac{x}{r} + r$$

$$\left(y = \frac{x}{r} + r \rightarrow x \in [-r, r] \right)$$

Arman

$$n^r + \varepsilon \rightarrow n \geq 1$$



$$y^r + \varepsilon = n \rightarrow y = \sqrt[n]{n - \varepsilon}$$

$$ry - 1 = n \rightarrow y = \frac{n}{r} + \frac{1}{r}$$

$$f^{-1}(n) = \begin{cases} \sqrt[n]{n - \varepsilon} & n \geq 8 \\ \frac{n+1}{3} & n \leq -1 \end{cases}$$

$$\frac{\cancel{n^r} + \cancel{r} \cancel{n^r} - \cancel{n^r} - 1 - \cancel{r} \cancel{n^r} - \cancel{r} \cancel{n}}{n + r} = \frac{-rn - 1}{n + r} = f(n) \quad -9$$

$$f^{-1}(n) \rightarrow \frac{-ry - 1}{y + r} = n \rightarrow -ry - 1 = ny + rn$$

$$(n + r)y = -rn - 1$$

$$f^{-1}(n) = \frac{-rn - 1}{n + r} = \frac{an + b}{rn + d} \rightarrow \begin{cases} d = 4 \\ a = -9 \\ b = -1 \end{cases}$$

$$f^{-1}(b) = \frac{-r(-r) - 1}{-r + r} = \boxed{a}$$

$$y = \frac{n}{n^r + 1} = f(n) \rightarrow \frac{y}{y^r + 1} = n \quad -1_0$$

$$ny^r + n = y \rightarrow ny^r + n - y = 0$$

$$y = \frac{1 \pm \sqrt{1 - \varepsilon n^r}}{r_n} \quad (I) \quad 1 - \varepsilon n^r \geq 0$$

$$1 \geq \varepsilon n^r \rightarrow \frac{1}{\varepsilon} \geq n^r$$

$$(II) \quad n \rightarrow \left[-\frac{1}{r}, \frac{1}{r}\right]$$

$$f^{-1}(n) \left\{ \begin{array}{ll} \frac{1 - \sqrt{1 - \varepsilon n^r}}{r_n} & 0 \leq n \leq \frac{1}{r} \\ \frac{1 + \sqrt{1 - \varepsilon n^r}}{r_n} & -\frac{1}{r} \leq n < 0 \end{array} \right.$$