



Date: / /

Subject: _____



تالیف II

نماذج

نیز حد اول را بیابید. یا از هم رفته

$$f(x) = \begin{cases} x^3 - 4, & x > a \\ 12x - 2, & x < a \end{cases} \quad \begin{aligned} & x^3 - 4 = 12x - 2 \\ & x^3 - 12x - 14 = 0 \end{aligned} \quad \boxed{a \geq -4} \quad -1$$

$$2(a-2)(a+4) \geq 0 \rightarrow \frac{-4}{2} \leq a \leq 2$$

$$f^{-1}(2) = 4, \quad f(x) = 3x + k \rightarrow D_{f^{-1}} = R_f, \quad D_f = R_{f^{-1}} \quad -2$$

$$3(4) + k = 2 \rightarrow k = -10 \quad f(x) = 3x - 10$$

$$\text{الف) } f(v) = 3 \times v - 10 = 11 \quad -11$$

$$\text{ب) } f(f(x)) = 3(3x - 10) - 10 = 9x - 40 \quad -40$$

$$f(a) = \frac{ax}{x-1}, \quad A(2a, a) \rightarrow f(a) \text{ در } A \quad -3$$

$$\frac{a^2}{a-1} = 2a \rightarrow a^2 = 2a^2 - 2a \rightarrow a^2 - 2a = 0 \rightarrow a(a-2) = 0$$

$$a = 2 \quad \checkmark \quad \text{اگر } a = 0 \text{ غلط است. چون تابع ثابت می باشد! و رابطه یک به یک نیست.}$$

$$\text{الف) } f \circ f^{-1} = \{(5, 5), (7, 7), (2, 2), (4, 4)\} \quad -4$$

$$\text{ب) } f^{-1} \circ f = \{(3, 3), (4, 4), (7, 7), (9, 9)\}$$

$$\text{ج) } f \circ g^{-1} = \{(2, 5), (5, 4)\} \quad 4 \rightarrow 1 \rightarrow X$$

$$\text{د) } g^{-1} \circ f = \{(2, 4), (7, 3), (9, 1)\}$$

$$g^{-1} = \{(1, 2), (3, 4), (5, 4)\} \quad f \circ g^{-1} = \{(1, 4), (3, 1), (5, 0)\} \quad -5$$

$$D_h \cap f \circ g^{-1} = \{1, 3, 5\}$$

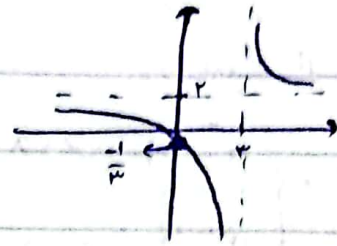
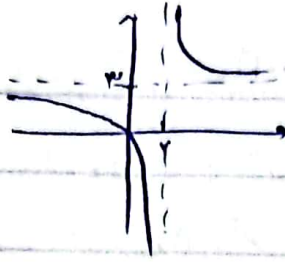
$$\frac{h}{f \circ g^{-1}} = \left\{ \left(1, \frac{2}{4}\right), \left(3, \frac{4}{1}\right), \left(5, \frac{0}{0}\right) \right\} = \left\{ \left(1, \frac{1}{2}\right), \left(3, \frac{1}{4}\right) \right\}$$



ریشه مخفی، جانب قائم

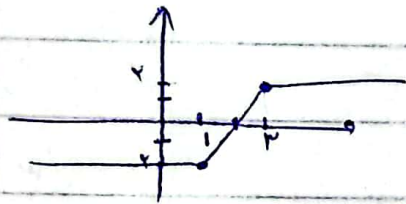
$\frac{a}{c}$ و جانب افقی

$$x \pm 1 \rightarrow y \pm \varepsilon$$



$$y = \frac{3x+1}{x-2} \rightarrow 3x+1 = yx-2y \rightarrow x(y-3) = 2y-1 \rightarrow x = \frac{2y-1}{y-3} = f^{-1}\left(\frac{2y+1}{y-3}\right)$$

$$f(x) = |x-1| - |x-3|, [a, b]$$



$$[1, 3] \text{ در } y, y = 2x - \varepsilon \rightarrow x = \frac{y + \varepsilon}{2}$$

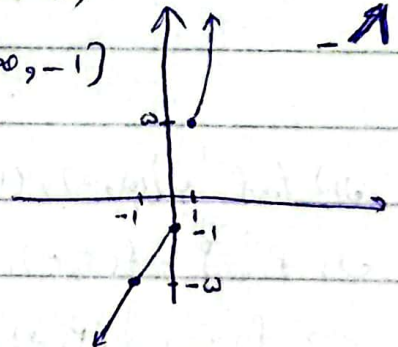
$$y = \frac{x + \varepsilon}{2} \rightarrow f^{-1}(a) = \frac{x + \varepsilon}{2}$$

$$f(x) = \begin{cases} x^3 + \varepsilon, & x \geq 1 \\ \varepsilon x - 1, & x \leq 0 \end{cases} \quad \begin{matrix} \frac{x+1}{y} \rightarrow R: [0, +\infty) \\ \frac{x}{y-1} \rightarrow R: (-\infty, -1) \end{matrix}$$

$$y = x^3 + \varepsilon \rightarrow x = \sqrt[3]{y - \varepsilon} \rightarrow f^{-1}(a) = \sqrt[3]{a - \varepsilon}$$

$$y = \varepsilon x - 1 \rightarrow x = \frac{y + 1}{\varepsilon} \rightarrow f^{-1}(a) = \frac{a + 1}{\varepsilon}$$

$$f^{-1}(x) = \begin{cases} \sqrt[3]{x - \varepsilon}, & x \geq 0 \\ \frac{x + \varepsilon}{\varepsilon}, & x \leq -1 \end{cases}$$



$$f(x) = x^3 - \frac{(x+1)^3}{x+3} \quad f^{-1}(a) = \frac{ax+b}{cx+d} \quad f^{-1}(b) = ?$$

$$\rightarrow = \frac{x^3 + 3x^2 - x^3 - 3x - 3x^2 - 1}{x+3} = \frac{-3x-1}{x+3} \rightarrow f(x) = \frac{-3x-1}{x+3}$$

$$y = \frac{-3x-1}{x+3} \rightarrow yx + 3y = -3x - 1 \rightarrow x = \frac{-3y-1}{y+3} \rightarrow f^{-1}(a) = \frac{-3a-1}{a+3}$$

$$\rightarrow a = -4, b = -2, a = 4 \rightarrow f^{-1}(-2) = \frac{-3(-2)-1}{-2+3} = 5$$

در 2 منفی

$$f(x) = \frac{x}{x^2+1} \rightarrow y = \frac{x}{x^2+1} \rightarrow yx^2 - x + y = 0 \rightarrow R: [-\frac{1}{2}, \frac{1}{2}]$$

$$\rightarrow \Delta = 1 - 4y^2 \geq 0 \rightarrow y \in [-\frac{1}{2}, \frac{1}{2}]$$

$$x = \frac{1 \pm \sqrt{1-4y^2}}{2y} \rightarrow y^{-1} = \frac{1 \pm \sqrt{1-4y^2}}{2y} \rightarrow f^{-1}\left(\frac{1 \pm \sqrt{1-4y^2}}{2y}\right) = \frac{1 \pm \sqrt{1-4y^2}}{2y}$$