

نام و نام خانوادگی: هلا علی

پاسخنامه تشریحی تکلیف شماره کلاس: (دست)

$$(x^3 - 4) \min \geq (12x - 5) \max \rightarrow x^3 - 4 - 12x + 5 \geq 0 \rightarrow x^3 - 12x + 1 \geq 0$$

$$x(x - 2)(x + 2) - 12(x - 2) \geq 0 \rightarrow (x - 2)(x(x + 2) - 12) \geq 0 \quad (x - 2)(x^2 + 2x - 12) \geq 0$$

$$\left\{ \begin{array}{l} x = 2 \\ x = -4 \end{array} \right. \quad \text{نقطه های بحرانی}$$

$$D_f = [-4, +\infty) \rightarrow 0x \geq -4$$

$$f(x) = 3x + k \quad f^{-1}(x) \Rightarrow x = 3y + k \xrightarrow{f^{-1}(y)} y = 12 + k \rightarrow \boxed{k = -12} \rightarrow f(x) = 3x - 12$$

$$f^{-1}(x) = 4$$

$$\text{الف)} \quad f(y) : f(y) = 3y - 12 = 11$$

$$\text{ب)} \quad f(f(x)) = f(3x - 12) - 12 = 9x - 36 - 12 = 9x - 48$$

$$f(x) = \frac{ax}{x-1} \xrightarrow{f^{-1}(y)} x = \frac{ay}{y-1} \quad A(50, 5) \xrightarrow{A(50, 5)} 50 = \frac{a(5)}{5-1} \Rightarrow \frac{50 \times 4}{4} = 50 = 50$$

$$a^2 - 5a = 0 \Rightarrow a(a - 5) = 0 \rightarrow \begin{cases} a = 0 \\ a = 5 \end{cases} \quad \text{نقطه } x \rightarrow y = 0$$

$$\rightarrow \boxed{a = 5}$$

$$\text{الف)} \quad f \circ f^{-1} : \{(5, 5), (5, 5), (5, 5)\}$$

$$\text{ب)} \quad f^{-1} \circ f : \{(5, 5), (5, 5), (5, 5)\}$$

$$\text{ج)} \quad f \circ g^{-1} : \{(5, 5)\}$$

$$\text{د)} \quad g^{-1} \circ f : \{(5, 5), (5, 5), (5, 5)\}$$

$$f \circ g^{-1} = \{(5, 5), (5, 5)\}$$

$$\frac{h}{f \circ g^{-1}} = \left\{ \left(1, \frac{1}{2}\right), \left(2, \frac{1}{4}\right), \left(3, \frac{1}{8}\right) \right\} \rightarrow \frac{1}{f \circ g^{-1}} = \left\{ \left(1, \frac{1}{2}\right), \left(2, \frac{1}{4}\right), \left(3, \frac{1}{8}\right) \right\}$$

تابع معکوس $y = \frac{3x+1}{x-2} \xrightarrow{\text{تابع معکوس}} \begin{cases} y = \frac{c}{a} = 3 \\ ad - bc = -4 - 1 < 0 \rightarrow \text{نزولی} \\ x = \frac{c}{a} = 3 \end{cases}$ مکمل $x = \frac{3y+1}{y-2} = x \Rightarrow y = -\frac{-3x-1}{x-3} \Rightarrow y = \frac{3x+1}{x-3}$ $y = \frac{c}{a} = 3$ $x = \frac{c}{a} = 3$ $ad - bc = -4 - 1 < 0$ نزولی مکمل معکوس	$y = \frac{3x+1}{x-2} \rightarrow x = \frac{3y+1}{y-2}$
تابع آمارتی $f(x) = x-1 - x-3 $ $R_f = [-2, 2]$ $D_f = [1, 3]$ بردار تابع، دامنه تابع معکوس است و دامنه تابع بردار تابع معکوس مکمل $f^{-1}(x) = \frac{x}{2} + 2, -2 \leq x \leq 2$	$\frac{1}{-2} \left \frac{x-1+x}{-2-x} = \frac{2}{x-3} \right$ $\Rightarrow y = 2x-4$ $x = 2y-4 \rightarrow y = \frac{x+4}{2}$
$R_f = [0, +\infty)$ $R_f = (-\infty, -1]$ تابع معکوس $f^{-1}(x) = \begin{cases} \sqrt{x-4} ; x \geq 4 \\ \frac{x}{x+1} + \frac{1}{x} ; x \leq -1 \end{cases}$	$y = \sqrt{x-4} \rightarrow x = y^2 + 4 \rightarrow y = \sqrt{x-4}$ $y = \frac{x}{x+1} \rightarrow x = \frac{y}{1-y} \rightarrow \frac{x+1}{y} = y \rightarrow y = \frac{x+1}{x}$ $D_f^{(-1)} = [0, +\infty)$ $D_f^{(-1)} = (-\infty, -1]$
تابع معکوس $f(x) = x^2 - \frac{(x+1)^2}{x+2} = \frac{x^2 + 2x^2 - x^2 + 5x^2 - 5x - 1}{x+2} = \frac{-5x-1}{x+2} \rightarrow y = \frac{-5x-1}{x+2}$ $f^{-1}(x) \rightarrow x = \frac{-5y-1}{y+2} \rightarrow y = -\frac{5x+1}{x+2} \rightarrow y = \frac{-5x-1}{x+2} \xrightarrow{\times \frac{x}{x}} y = \frac{-5x-1}{x+2}$ $f^{-1}(x) = \frac{ax+b}{cx+d} = \frac{-5x-1}{x+2} \rightarrow \begin{cases} a=-5 \\ b=-1 \\ c=1 \\ d=2 \end{cases} \rightarrow f^{-1}(b) = f^{-1}(-1) = \frac{-5(-1)-1}{-1+2} = \frac{4}{1} = 4$	9
$f(x) = \frac{x}{x^2+1} \rightarrow f^{-1}(x) : x = \frac{y}{y^2+1} \rightarrow xy^2 - y + x = 0$ $\Delta = 1 - 4x^2 \geq 0 \rightarrow \frac{1}{4} \geq x^2 \rightarrow \frac{1}{2} \geq x \geq -\frac{1}{2} : I$ $f^{-1}(x) = \frac{1 \pm \sqrt{1-4x^2}}{2x}, x \in \left(\left[-\frac{1}{2}, \frac{1}{2} \right] - \{0\} \right)$	$\frac{x = \frac{b \pm \sqrt{b^2 - 4ac}}{2a}}{b = 2x, c = 1} \rightarrow y = \frac{1 \pm \sqrt{1-4x^2}}{2x}$ $2x \neq 0 : II$ $D_f : I \cap II : \left[-\frac{1}{2}, \frac{1}{2} \right] - \{0\}$ 10