

$$(x^3 - 4) \min \geq (12x - 5) \max \rightarrow x^3 - 4 - 12x + 5 \geq 0 \rightarrow x^3 - 12x + 1 \geq 0$$

$$x(x - 2)(x + 2) - 12(x - 2) \geq 0 \rightarrow (x - 2)(x(x + 2) - 12) \geq 0 \quad (x - 2) \left(\frac{x^2 + 2x - 12}{(x + 2)(x - 2)} \right) \geq 0$$

$$\left\{ \begin{array}{l} x = 2 \\ x = -4 \end{array} \right. \quad \text{نقطه های بحرانی}$$

$$D_f : [-4, +\infty) \rightarrow 0 \geq -4 \quad (5)$$

$$f(x) = 3x + k \quad f^{-1}(x) \Rightarrow x = 3y + k \xrightarrow{f^{-1}(y)} y = 12 + k \rightarrow \boxed{k = -12} \rightarrow f(x) = 3x - 12$$

$$f^{-1}(x) = 4$$

$$\text{الف) } f(y) : f(y) = 3y - 12 = 11 \quad (5)$$

$$\text{ب) } f(f(x)) = f(3x - 12) = 3(3x - 12) - 12 = 9x - 36 - 12 = 9x - 48$$

$$f(x) = \frac{ax}{x-1} \xrightarrow{f^{-1}(x)} x = \frac{ay}{y-1} \quad A(20, 5) \xrightarrow{A(20, 5)} 20 = \frac{a(5)}{5-1} \Rightarrow \frac{20 \times 4}{5} = 16 = a$$

$$a^2 - 5a = 0 \Rightarrow a(a - 5) = 0 \rightarrow \begin{cases} a = 0 \\ a = 5 \end{cases} \quad \text{نقطه } x \rightarrow y = 0 \quad \text{نقطه } y = 0 \rightarrow x = 0$$

$$\text{الف) } f \circ f^{-1} : \{(5, 20), (20, 5)\}$$

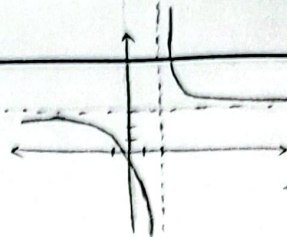
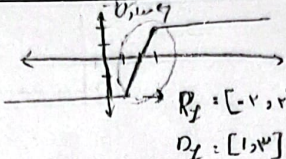
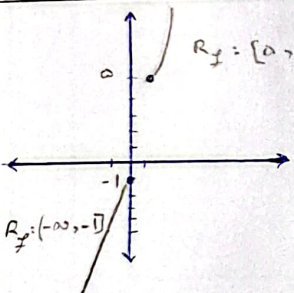
$$\text{ب) } f^{-1} \circ f : \{(5, 20), (20, 5)\}$$

$$\text{ج) } f \circ g^{-1} : \{(5, 20), (20, 5)\}$$

$$\text{د) } g^{-1} \circ f : \{(5, 20), (20, 5)\}$$

$$f \circ g^{-1} = \{(5, 20), (20, 5)\}$$

$$\frac{h}{f \circ g^{-1}} = \left\{ \left(1, \frac{1}{f} \right), \left(\frac{1}{f}, 1 \right) \right\} \rightarrow \frac{1}{f} = \left\{ \left(1, \frac{1}{f} \right), \left(\frac{1}{f}, 1 \right) \right\}$$

تابع معکوس $y = \frac{3x+1}{x-2} \xrightarrow{\text{تابع معکوس}} \begin{cases} y = \frac{c}{a} = 3 \\ ad - bc = -4 - 1 < 0 \rightarrow \text{نزولی} \\ x = \text{میانگین} = \frac{a+d}{2} = 1 \end{cases}$ مکمل $\lambda = \frac{3y+1}{y-2} = x \Rightarrow y = -\frac{-3x-1}{x-3} \Rightarrow y = \frac{3x+1}{x-3}$ $y = \frac{c}{a} = 3$ $x = \text{میانگین} = \frac{a+d}{2} = 1$ $ad - bc = -4 - 1 < 0$ نزولی  $y=3 \Rightarrow x=1$ (میانگین)	5
تابع آمارتی $f(x) = x-1 - x-3 $  $R_f = [-2, 2]$ $D_f = [1, 3]$ بردار تابع، دامنه تابع معکوس است و دامنه تابع بر دانه مکمل $f^{-1}(x) = \frac{x}{2} + 2, -2 \leq x \leq 2$	5 $\frac{1}{-2} \left \frac{x-1+x}{-2} \right \frac{2}{-2} = \frac{x-1}{-2}$ $\Rightarrow y = 2x - 4$ $x = 2y - 4 \rightarrow y = \frac{x+4}{2}$ 7
تابع معکوس  $R_f = [0, +\infty)$ $R_f^{-1} = (-\infty, -1]$ $f^{-1}(x) = \begin{cases} \sqrt{x-4}; x \geq 4 \\ \frac{x}{4} + \frac{1}{4}; x \leq -1 \end{cases}$	$y = x^2 + 4 \rightarrow x = \sqrt{y-4} \rightarrow y = \sqrt{x-4}$ $y = 4x - 1 \rightarrow x = \frac{y+1}{4} \rightarrow \frac{x+1}{4} = y$ $D_f^{-1} = (-\infty, -1]$ 1
تابع معکوس $f(x) = x^2 - \frac{(x+1)^2}{x+2} = \frac{x^2 + 2x^2 - x^2 + 2x^2 - 2x - 1}{x+2} = \frac{-2x-1}{x+2} \rightarrow y = \frac{-2x-1}{x+2}$ $f^{-1}(x) \rightarrow x = \frac{-2y-1}{y+2} \rightarrow y = -\frac{2x+1}{x+2} \rightarrow y = \frac{-2x-1}{x+2} \xrightarrow{\times \frac{x}{x}} y = \frac{-4x-2}{2x+4}$ $f^{-1}(x) = \frac{ax+b}{cx+d} = \frac{-4x-2}{2x+4} \rightarrow \begin{cases} a=-4 \\ b=-2 \\ c=2 \\ d=4 \end{cases} \rightarrow f^{-1}(b) = f^{-1}(-2) = \frac{-2(-2)-1}{-2+2} = \frac{3}{0} = \infty$	5 9
$f(x) = \frac{x}{x^2+1} \rightarrow f^{-1}(x): x = \frac{y}{y^2+1} \rightarrow xy^2 - y + x = 0$ $\Delta = 1 - 4x^2 \geq 0 \rightarrow \frac{1}{4} \geq x^2 \rightarrow \frac{1}{2} \geq x \geq -\frac{1}{2} : I$ $f^{-1}(x) = \frac{1 \pm \sqrt{1-4x^2}}{2x}, x \in \left(\left[-\frac{1}{2}, \frac{1}{2} \right] - \{0\} \right)$	$\frac{x = \frac{b \pm \sqrt{b^2 - 4ac}}{2a}}{b = 2x - 4ac} \rightarrow y = \frac{1 \pm \sqrt{1-4x^2}}{2x}$ $D_f: I \cap II: \left[-\frac{1}{2}, \frac{1}{2} \right] - \{0\}$ 10