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$$f(n) = \begin{cases} n^3 - 4 & ; n > a \\ 12n - 20 & ; n \leq a \end{cases} \rightarrow \text{مردمانه مقدار} \rightarrow n^3 - 4 > 12n - 20$$

$$\rightarrow \text{کمترین مقدار} = a^3 - 4$$

$$12n - 20 = 12a - 20$$

$$a^3 - 4 \geq 12a - 20 \rightarrow a^3 - 12a + 16 \geq 0 \rightarrow a(a^2 - 12) + 16 \geq 0$$

$$\Rightarrow (a-2)(a^2 + 2a - 1) \geq 0 \rightarrow (a-2)(a+1) \geq 0$$

$$\Rightarrow a \in [-1, +\infty)$$

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$$f(n) = 3n + k$$

$$f^{-1}(2) = 4 \rightarrow f(4) = 2$$

$$f(n) = 3n - 10$$

$$f(v) = 3v - 10 = 11$$

$$f(f(n)) = 3(3n - 10) - 10 = 9n - 40$$

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$$f(n) = \frac{an}{n-1}$$

$$A(a, a) \rightarrow f^{-1}(a) = a \rightarrow f(a) = a$$

$$a = \frac{a^2}{a-1} \rightarrow a^2 - a = a^2$$

$$\Rightarrow a^2 - a = 0 \rightarrow a(a-1) = 0$$

چون تابع یک به یک نیستی شود (سبب نه تابع نام-یک است)

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$$f = \{(3, 5), (4, 6), (5, 7), (6, 8)\} \rightarrow f^{-1} = \{(5, 3), (6, 4), (7, 5), (8, 6)\}$$

$$g = \{(3, 2), (4, 3), (5, 4), (6, 5)\} \rightarrow g^{-1} = \{(2, 3), (3, 4), (4, 5), (5, 6)\}$$

$$f \circ f^{-1} = \{(5, 5), (6, 6), (7, 7), (8, 8)\}$$

$$f^{-1} \circ f = \{(3, 3), (4, 4), (5, 5), (6, 6)\}$$

$$f \circ g^{-1} = \{(2, 5), (3, 6), (4, 7), (5, 8)\}$$

$$g^{-1} \circ f = \{(3, 6), (4, 7), (5, 8), (6, 9)\}$$

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$$f = \{(2, 4), (3, 8), (4, 0)\}$$

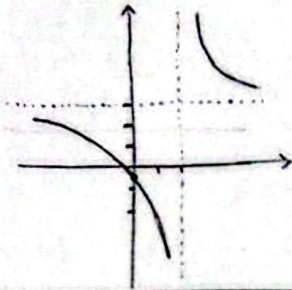
$$g = \{(2, 1), (3, 2), (4, 5)\} \rightarrow g^{-1} = \{(1, 2), (2, 3), (5, 4)\}$$

$$h = \{(1, 2), (2, 4), (3, 1), (4, 1)\}$$

$$h \circ f^{-1} = \{(1, \frac{1}{4}), (2, \frac{1}{2}), (3, \frac{1}{8})\}$$

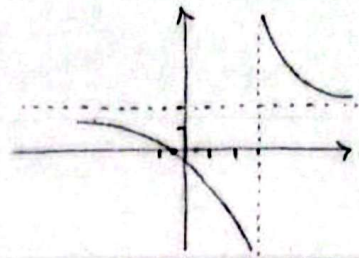


$$y = \frac{x+1}{x-2}$$



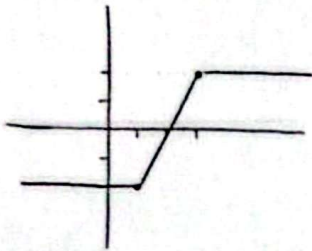
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$$f^{-1}(x): x = \frac{y+1}{y-2} \rightarrow y = \frac{x+1}{x-2}$$



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$$f(x) = |x-1| - |x-2|$$



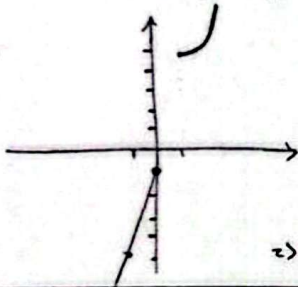
$$[a, b] = [1, 2] \rightarrow f(x) = x-2; x \geq 2$$

$$f^{-1}(x): x = y-2 \rightarrow y = \frac{x+2}{1} \rightarrow \frac{x+2}{1} \geq 2 \rightarrow x \geq 0$$

$$y = \frac{x+2}{1} \rightarrow x \leq 2$$

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$$f(x) = \begin{cases} x^2+1 & x \geq 1 \\ x-1 & x \leq 0 \end{cases}$$



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$$f(x) = x^2+1 \rightarrow D_f = [1, +\infty) \\ R_f = [2, +\infty)$$

$$f^{-1}(x): x = y^2+1 \rightarrow y = \sqrt{x-1}; x \geq 2$$

$$f(x) = x-1 \rightarrow D_f = (-\infty, 0] \\ R_f = (-\infty, -1]$$

$$x = y-1 \rightarrow y = \frac{x+1}{1}; x \leq -1$$

$$f^{-1}(x) = \begin{cases} \sqrt{x-1}; x \geq 2 \\ \frac{x+1}{1}; x \leq -1 \end{cases}$$

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$$f(x) = x^2 - \frac{(x+1)^2}{x+2} = \frac{x^3 + 2x^2 - x^2 - 1 - 2x^2 - 2x}{x+2} = \frac{-x-1}{x+2}$$

$$y = \frac{-x-1}{x+2} \xrightarrow{\text{cross}} x = \frac{-y-1}{y+2} \rightarrow y = \frac{-x-1}{x+2} = \frac{-4x-4}{x+4}$$

$$f^{-1}(x) = \frac{ax+b}{cx+d} \rightarrow \frac{ax+b}{cx+d} = \frac{-4x-4}{x+4} \rightarrow a=-4, b=-4, c=1, d=4$$

$$f^{-1}(x) = \frac{-x-1}{x+2} \rightarrow f^{-1}(x) = \frac{-x-1}{x+2} = \frac{-x-1}{x+2}$$

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$$f(x) = \frac{x}{x^2+1} \rightarrow yx^2 - x + y = 0 \rightarrow x = \frac{1 \pm \sqrt{1-4y^2}}{2y} \xrightarrow{\text{positive}} x = \frac{1 + \sqrt{1-4y^2}}{2y}$$

$$f^{-1}(x): y = \frac{1 + \sqrt{1-4x^2}}{2x}; \frac{1}{2} \leq x \leq \frac{1}{2}, x \neq 0$$

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