

$$f(x) = \begin{cases} x^2 - 4 & ; x > a \\ 12x - 10 & ; x \leq a \end{cases}$$

$$a^2 - 4 \geq 12a - 10 \Rightarrow a^2 - 12a + 14 \geq 0 \Rightarrow a^2 - 12a + 14 \geq 0 \Rightarrow a(a-12) + 4(a-2) \geq 0$$

$$\Rightarrow (a+4)(a^2 - 12a + 4) \geq 0 \Rightarrow (a+4)(a-2)^2 \geq 0 \Rightarrow a \geq 2$$

پایه یک، دو، سه، چهار

$$f(x) = 12x + 10 \xrightarrow{x=10} 12 \cdot 10 + 10 = 130 \rightarrow f(x) = 12x - 10$$

$$f^{-1}(x) = \frac{x+10}{12}$$

الف) $f(10) = 12 \cdot 10 - 10 = 110$

ب) $f(f(x)) = f(12x - 10) = 12(12x - 10) - 10 = 144x - 130 = 144x - 130$

داده: $A(x, y) \rightarrow B(x, y)$

$$f(x) = \frac{ax}{x-1} \xrightarrow{x=a} \frac{a^2}{a-1} = 2a \Rightarrow 2a^2 - 2a = a^2 \Rightarrow a^2 - 2a = 0 \Rightarrow a(a-2) = 0$$

$a=0$ $a=2$

$f = \{(1, 0), (2, 1), (3, 2), (4, 3)\}$ $g = \{(1, 2), (2, 4), (3, 6)\}$ $f^{-1} = \{(0, 1), (1, 2), (2, 3), (3, 4)\}$

الف) $f \circ f^{-1} = \{(1, 1), (2, 2), (3, 3), (4, 4)\}$ $g^{-1} = \{(2, 1), (4, 2), (6, 3)\}$

ب) $f^{-1} \circ f = \{(1, 1), (2, 2), (3, 3), (4, 4)\}$

ج) $g \circ g^{-1} = \{(2, 2), (4, 4), (6, 6)\}$

د) $g^{-1} \circ f = \{(1, 2), (2, 4), (3, 6)\}$

$f = \{(1, 2), (2, 1), (3, 0)\}$ $g = \{(1, 1), (2, 2), (3, 3)\}$ $h = \{(1, 1), (2, 2), (3, 3), (4, 4)\}$

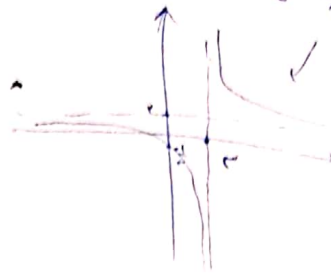
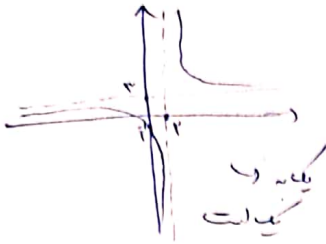
$h \circ g^{-1} = \{(1, 1), (2, 2), (3, 3), (4, 4)\}$

$g^{-1} = \{(1, 1), (2, 2), (3, 3)\}$

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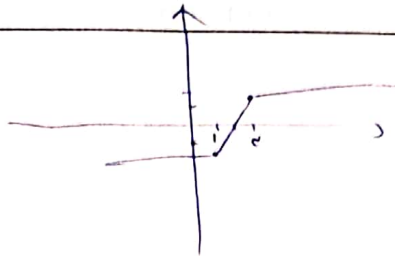
$h \circ g^{-1} = \{(1, 1), (2, 2), (3, 3), (4, 4)\}$

$$f(x) = \frac{x+1}{x-1} \xrightarrow{\text{sub}} a = \frac{y+1}{y-1} \rightarrow xy - yn = y+1 \rightarrow y(n-1) = y+1 \rightarrow y = \frac{n+1}{n-1}$$



$$f(x) = |x-1| - |x-1|$$

$$\begin{aligned} & \begin{cases} 1-x+n & |x-1| & n-1-n+1 & \\ = -x & = x-1 & = 0 & \end{cases} \end{aligned}$$

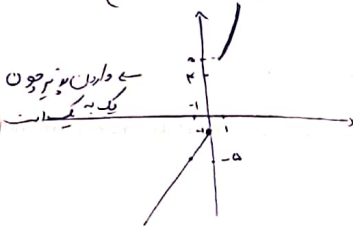


$$\rightarrow \text{sign} \quad \begin{cases} [1, \infty) \\ \text{increasing} \end{cases}$$

$$\rightarrow y = x-1 \xrightarrow{\text{sub}} n = y+1 \rightarrow y = \frac{n+1}{n}$$

$$f(x) = \begin{cases} x+1 & ; x < 1 \\ x-1 & ; x \geq 1 \end{cases} \xrightarrow{\text{sub}} a = y+1 \rightarrow y = x-1$$

$$\rightarrow f^{-1}(x) = \begin{cases} x-1 & ; x \geq 1 \\ \frac{x+1}{2} & ; x < 1 \end{cases}$$



$$f(x) = a - \frac{(n+1)^2}{n+1} = a - \frac{n^2 + 2n + 1}{n+1} = a - \frac{n^2 + 2n + 1}{n+1} \xrightarrow{\text{sub}} a = \frac{-y-1}{y+1} \rightarrow xy + yn = -y-1$$

$$f^{-1}(x) = \frac{ax+b}{cx+d} \rightarrow y(n+1) = \frac{-1-yn}{-1+y} \Rightarrow y = \frac{-1-yn}{-1+y}$$

$$\frac{ax+b}{cx+d} = \frac{-1-yn}{-1+y} \quad f^{-1}(b) = f^{-1}(-1) = \frac{-1-(-1)}{-1+1} = \frac{0}{0} = \text{undefined}$$

$$\begin{aligned} a &= -4 \\ b &= -1 \\ d &= 4 \end{aligned} \rightarrow \frac{-4n-1}{n+4} = \frac{-4(n+1)+3}{n+4}$$

$$f(x) = \frac{a}{x+1} \xrightarrow{\text{sub}} a = \frac{y}{y+1} \rightarrow xy + x + y \rightarrow xy - y + a = 0$$

$$y = \frac{1 \pm \sqrt{1-4ax}}{2a} \quad \begin{aligned} & \text{if } a > 0 \\ & \text{if } a < 0 \end{aligned} \quad \begin{aligned} f^{-1}(x) &= \frac{1 + \sqrt{1-4ax}}{2a} \\ f^{-1}(x) &= \frac{1 - \sqrt{1-4ax}}{2a} \end{aligned}$$