(1

$$f(x)\begin{cases} x^2-4 & x>a \\ 12x-20 & x\leq a \end{cases}$$

$a^2-4 \geq 12a-20$

$a^2-12a+16 \longrightarrow (a-2)^3+6(a^2-4a+4)>0$

$(a-2)^2+6(a-2)^2 \Rightarrow (a-2)^2(a-2+6)>0$

$\longrightarrow (a-2)^2(a+4)>0 \longrightarrow \begin{array}{c|c|c} & -4 & 2 \\ - & + & + \end{array} \longrightarrow a \geq -4$

(2

$f(x)=3x+k \longrightarrow f^{-1}(x) \Rightarrow 3y+k=x \longrightarrow y=\frac{x-k}{3}$

$f^{-1}(2)=4 \longrightarrow \frac{2-k}{3}=4 \longrightarrow 2-k=12 \longrightarrow k=-10 \longrightarrow f(x)=3x-10$

الف) $f(7)=3\times 7-10=\boxed{11}$ ب) $f(f(x))=3(3x+k)+k=9x+4k=\boxed{9x-40}$

(3

$f(x)=\frac{ax}{x-1} \longrightarrow x=\frac{ay}{y-1} \longrightarrow xy-x=ay \longrightarrow xy-ay=x$

$y(x-a)=x \longrightarrow y=\frac{x}{x-a}$

$a=\frac{2a}{\frac{2a-a}{a}} \longrightarrow a^2=2a \longrightarrow a^2-2a=0$

$a(a-2) \longrightarrow a=0$ غ ق ق ، $\boxed{a=2}$

(4

$f(x)=\{(3,5)(4,7)(7,2),(9,9)\} \quad g=\{(3,2)(8,6)(9,5)\}$

الف) $f\circ f^{-1} \longrightarrow f^{-1}(x)=\{(5,3)(7,4)(2,7)(9,9)\}$

$f\circ f^{-1}=\{(5,5)(7,7),(2,2)(9,9)\}$ $\left| \begin{array}{l} R_{f^{-1}}=D_f \\ D_{f^{-1}}=R_f \end{array} \right| \rightsquigarrow D_{f\circ f^{-1}}=R_{f\circ f^{-1}}$

ب) $f^{-1}\circ f \longrightarrow ?=\{(3,3)(4,4)(7,7)(9,9)\}$

ج) $f\circ g^{-1} \longrightarrow g^{-1}(x)=\{(2,3)(6,8)(5,9)\} \longrightarrow f\circ g^{-1}=\{(2,5)(5,9)\}$

د) $g^{-1}\circ f \longrightarrow g^{-1}\circ f=\{(3,9)(7,3)(9,8)\}$

(5

$f=\{(2,4),(4,8),(6,0)\} \quad g=\{(2,1)(4,3)(6,5)\} \quad h=\{(1,2),(3,4),(5,1)\}$

$f\circ g^{-1}= \quad g^{-1}=\{(1,2)(3,4)(5,6)\}$

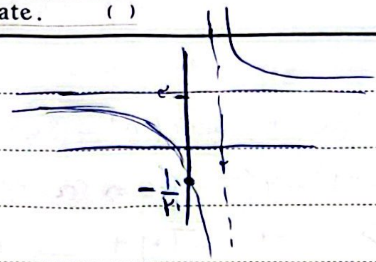
$f\circ g^{-1}=\{(1,4)(3,8)(5,0)\}$

$\frac{h}{f\circ g^{-1}}=\{(1,\frac{1}{2})(3,\frac{1}{2})\cancel{(5,\frac{1}{0})}\}$ غ ق ق

$\frac{h}{f\circ g^{-1}}=\{(1,\frac{1}{2})(3,\frac{1}{2})\}$

۶)

$y = \dfrac{3x+1}{x-2}$ $\xrightarrow{\text{حوزه اش}}$ $x-2=0 \to x=\boxed{2}$, $\dfrac{3x}{x} = \boxed{3}$



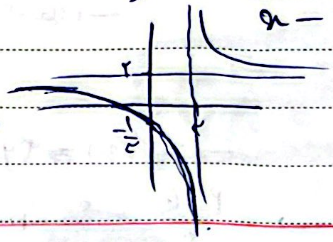
→ یک به یک است → $x = \dfrac{3y+1}{y-2} = x$

$y = \dfrac{-2x-1}{x-3} = \boxed{\dfrac{2x+1}{x-3}}$

$\dfrac{2x+1}{x-3}$

$x-3=0 \Rightarrow x=3$

$\dfrac{2x}{x} = 2$



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۷)

$f(x) = |x-1| - |x-3|$, $[a\ b] = [1\ 3]$

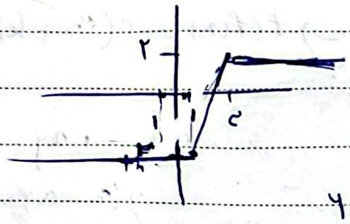
ریشه ها

$x-1=0 \to x=1$

$x-3=0 \to x=3$

$|1-1| - |1-3| = -2$

$|3-1| - |3-3| = 2$



$f(x) = \begin{array}{c|c|c} & 1 & 3 \\ -2 & 2x-4 & 2 \end{array}$

$y = 2x-4 \to x = 2y-4 \to y = \dfrac{x+4}{2}$

$\boxed{y = \frac{1}{2}x + 2}$

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۸)

$f(x)\begin{cases} x^2+4 & x \geq 1 \to R = [5\ +\infty) \\ 4x-1 & x \leq 0 \to R = (-\infty\ -1] \end{cases}$ یک به یک است

$\to x = y^2 - 4 \to y = \sqrt{x+4}$ $\quad x \geq 5$

$\to x = 4y-1 \to \dfrac{x+1}{4} \to y = \frac{1}{4}x + \frac{1}{4}$ $\quad x \leq -1$

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۹)

$f(x) = \dfrac{x^2(x+2) - (x+1)^2}{x+2} = \dfrac{-1-3x}{x+2} \to f^{-1}(x) \Rightarrow x = \dfrac{-1-3y}{y+2}$

$\dfrac{-1-3y}{y+2} = x \longrightarrow y = -\dfrac{3x+1}{x+3} \to y = \dfrac{-3x-1}{x+3} = \dfrac{ax+b}{2x+d}$

$\dfrac{-6x-2}{2x+6} = \dfrac{ax+b}{2x+d}$ $\begin{cases} a = -6 \\ b = -2 \\ d = 6 \end{cases}$

$f^{-1}(-2) = \dfrac{-3(-2)-1}{-2+3} = 5$

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۱۰)

$f(x) = \dfrac{x}{x^2+1} \to x = \dfrac{y}{y^2+1} \to xy^2 + x = y \to xy^2 - y + x = 0$

$\Delta = 1 - 4x^2 \to y = \dfrac{1 \pm \sqrt{1-4x^2}}{2x}$

$1 - 4x^2 \geq 0 \to$ $\begin{array}{ccccc} & -\frac{1}{2} & & \frac{1}{2} & \\ - & 0 & + & 0 & - \end{array}$

(چون x و y هم علامت هستند + قابل قبول است)

$y = \dfrac{1 + \sqrt{1-4x^2}}{2x} \longrightarrow D_{f^{-1}} = [-\frac{1}{2}\ \ \frac{1}{2}] - \{0\}$