(۱۱)

۱۹,۷۵

دریا دیانی دودستی

Subject :
Year . Month . Date . ( )

۱) $f(x)\begin{cases} x^2-4 & x>a \\ 12x-20 & x\leq a \end{cases}$

$a^2-4 \geq 12a-20$

$a^2-12a+16 \rightarrow (a-2)^2+4(a^2-4a+4)>0$

$(a-2)^2+4(a-2)^2 \Rightarrow (a-2)^2(a-2+4)>0$

$\rightarrow (a-2)^2(a+4)>0 \rightarrow$ $-4 \quad 2$ / $- \mid + \mid +$ $\rightarrow$ $a \geq -4$

۲) $f(x)=3x+k \rightarrow f^{-1}(x) \Rightarrow 3y+k=x \rightarrow y=\frac{x-k}{3}$

$f^{-1}(2)=4 \rightarrow \frac{2-k}{3}=4 \rightarrow 2-k=12 \rightarrow k=-10 \rightarrow f(x)=3x-10$

الف) $f(7)=3\times 7-10=11$ ب) $f(f(x))=3(3x+k)+k=9x+4k=9x-40$

۳) $f(x)=\frac{ax}{x-1} \rightarrow x=\frac{ay}{y-1} \rightarrow xy-x=ay \rightarrow xy-ay=x$

$y(x-a)=x \rightarrow y=\frac{x}{x-a}$

$a=\frac{2a}{\frac{2a}{a}-a} \rightarrow a^2=2a \rightarrow a^2-2a=0$

$a(a-2)$ → $a=0$ غ ق ق ، → $a=2$

۴) $f(x)=\{(3,5),(4,7),(7,2),(9,6)\}$ $g=\{(3,2),(8,6),(9,5)\}$

الف) $fof^{-1} \rightarrow f^{-1}(x)=\{(5,3),(7,4),(2,7),(6,9)\}$ $\quad R_{f^{-1}}=D_f$, $D_{f^{-1}}=R_f \rightarrow D_{fof^{-1}}=R_{fof^{-1}}$

$fof^{-1}=\{(5,5),(7,7),(2,2),(6,6)\}$

ب) $f^{-1}of \rightarrow =\{(3,3),(4,4),(7,7),(9,9)\}$

ج) $fog^{-1} \rightarrow g^{-1}(x)=\{(2,3),(6,8),(5,9)\} \rightarrow fog^{-1}=\{(2,5),(5,6)\}$

د) $g^{-1}of \rightarrow g^{-1}of=\{(3,9),(7,3),(9,8)\}$

۵) $f=\{(3,4),(4,8),(6,0)\}$ $g=\{(2,1),(4,3),(6,5)\}$ $h=\{(1,2),(3,4),(5,1)\}$

$fog^{-1}=$ $g^{-1}=\{(1,2),(3,4),(5,6)\}$

$fog^{-1}=\{(1,4),(3,8),(5,0)\}$

$\frac{h}{fog^{-1}}=\{(1,\frac{1}{2}),(3,\frac{1}{2}),\cancel{(5,\frac{1}{0})}\}$ غ ق ق

$\frac{h}{fog^{-1}}=\{(1,\frac{1}{2}),(3,\frac{1}{2})\}$

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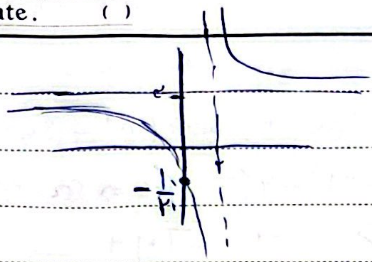
$y = \frac{3x+1}{x-2}$; $x-2=0 \to x=2$; $\frac{3x}{x} = 3$



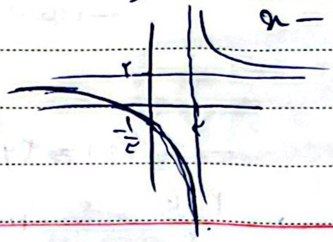
→ یک به یک است → $x = \frac{3y+1}{y-2} = x$

$y = \frac{-2x-1}{x-3} = \frac{2x+1}{x-3}$

$\frac{2x+1}{x-4}$

$x-4=0 \Rightarrow x=4$

$\frac{2x}{x} = 2$



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$f(x) = |x-1| - |x-3|$

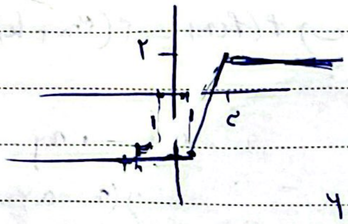
تابع آبشاری

$x-1=0 \to x=1$

$x-3=0 \to x=3$

$|1-1| - |1-3| = -2$

$|3-1| - |3-3| = 2$



$[a\ b] = [1\ 3]$

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$f(x) = $ $-2 \,\big|_{1}\, 2x-4 \,\big|_{3}\, 2$

$y = 2x-4 \to x = 2y-4 \to y = \frac{x+4}{2}$

$y = \frac{1}{2}x + 2$

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$f(x) \begin{cases} x^2+4 & x \geq 1 \to R = [5\ +\infty) \\ 4x-1 & x \leq 0 \to R = (-\infty\ -1] \end{cases}$ یک به یک است

$\to x = y^2 - 4 \to y = \sqrt{x-4}$ $\quad x \geq 5$

$\to x = 4y-1 \to \frac{x+1}{4} \to y = \frac{1}{4}x + \frac{1}{4}$ $\quad x \leq -1$

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$f(x) = \frac{x^2(x+3) - (x+1)^3}{x+3} = \frac{-1-3x}{x+3} \to f^{-1}(x) \Rightarrow x = \frac{-1-3y}{y+3}$

$\frac{-1-3y}{y+3} = x \to y = -\frac{3x+1}{x+3} \to y = \frac{-3x-1}{x+3} = \frac{ax+b}{2x+d}$

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$\frac{-6x-2}{2x+6} = \frac{ax+b}{2x+d}$ $\to a=-6,\ b=-2,\ d=6$

$f^{-1}(-2) = \frac{-3(-2)-1}{-2+3} = 5$

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$f(x) = \frac{x}{x^2+1} \to x = \frac{y}{y^2+1} \to xy^2 + x = y \to xy^2 - y + x = 0$

$\Delta = 1 - 4x^2 \to y = \frac{1 \pm \sqrt{1-4x^2}}{2x}$

$1 - 4x^2 \geq 0 \to$ $-\frac{1}{2}$, $\frac{1}{2}$: $-\ +\ -$

(چون برد هم علامت مثبت + قابل قبول است)

$y = \frac{1+\sqrt{1-4x^2}}{2x} \to D_{f^{-1}} = [-\frac{1}{2}\ \frac{1}{2}] - \{0\}$

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