(۵

$f=\{(2,4),(4,8),(6,0)\}$ $g=\{(2,1),(4,3),(6,5)\}$

$h=\{(1,2),(3,4),(4,1),(5,1)\}$ $\frac{h}{f o g^{-1}}=?$ $g^{-1}\{(1,2)(3,4)(5,6)\}$

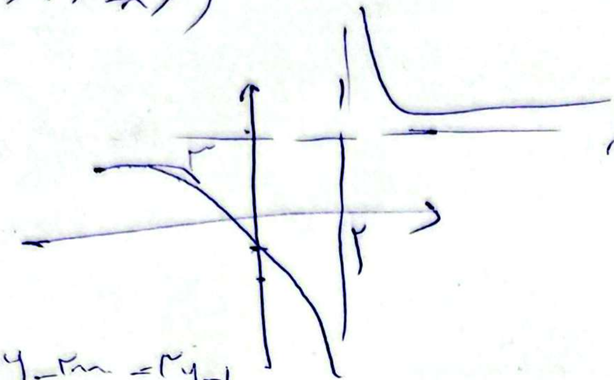
$f(g)^{-1}$ $1\to 2\to 4$ $5\to 6\to 0$
$3\to 4\to 8$ $f(g^{-1})=\{(1,4),(3,8)(5,0)\}$

۲

$\frac{h}{f o g^{-1}}=\{(1,\frac{2}{4}),(3,\frac{4}{8})\}$

(۶

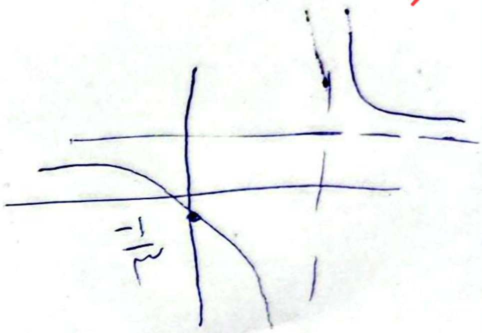
$y=\frac{3x+1}{x-2}$



$x=0\to y=-\frac{1}{2}$

۲

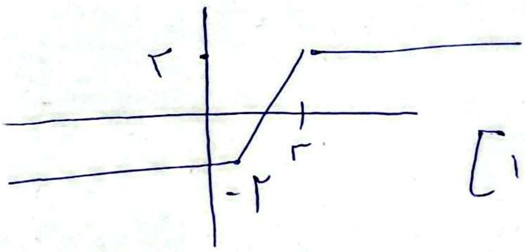
$x=\frac{2y+1}{y-3}\to xy-2x=3y-1$
$xy+3y=-1+2x$ $y=\frac{1+2x}{x-3}$



$f(x)=|x-1|-|x-3|$

(۷

$x+1+x-3=-2$ $x\le 1$



$[1,3]$ دامنه متغیر

$x-1+x-3=2x-4$ $1<x\le 3$

$x-1-x+3=2$ $x\ge 3$

۲

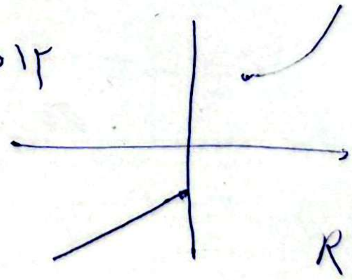
$1\le x\le 3\to$

$1\le x\le 3\to -x-1+x-3=2x-4$

$y=2x-4\to x=2y-4y$ $y=\frac{x+4}{2}$

۸

$f(x)=\begin{cases}x^3+4 & x\ge 1\to 1\to 5\ \ 2\to 12\\ 2x-1 & x\le 0\ \ -1-11\to\end{cases}$



یک به یک

۲

$y=x^3+4\to x=y^3+4$
$x-4=y^3\to y=\sqrt[3]{x-4}$

$R_{f(x)}=D_{f(x)^{-1}}$
$[5,+\infty)\cup(-\infty,-1]$

$f^{-1}(x)=\begin{cases}\sqrt[3]{x-4}\ ; & 5\le x\\ \frac{x+1}{2} & x\le -1\end{cases}$

۱۹,۷۵

$$f(x)\begin{cases} x^3-4 \;; & x>a \\ 12x-20 \;; & x\leq a \end{cases}$$

$12x-20 \leq x^3-4$
$12a-20 \leq a^3-4 \;\}\; a=x$

$a^3-12x+16\geq 0 \to a^3-12a+16 \;\; \lfloor a-2$
a^2+2a-8

$\dfrac{(a^2+2a-8)(a-2)}{(a+4)(a-2)} \geq 0$

$-4 \quad 2$
$- \;|\; + \;|\; +$ $\;\;[-4,+\infty)$ $\;\; D_f = R_{f^{-1}}$

$D_f = R_{f^{-1}}$

۳

سوال ۲:

الف) $f(7)$ $\quad f(x)=3x+n \quad f(2)=-4$

$f(7)=3\times7-10=11 \quad f(2)=-4 \quad 3\times2+n=-4 \to n=-10$

۲

ب) $f(f(x))=f(3x-10)=3(3x-10)-10 \to 9x-30-10=9x-40$

سوال ۳: $D_f=R_{f^{-1}}$

$f(x)=\dfrac{ax}{x-1} \quad f^{-1}(2a)=a \quad D_f^{-1}=R_f$

۱,۷۵

$f(a)=2a \to f(a)=\dfrac{a^2}{a-1}=2a \to 2a^2-2a=a^2$

$a^2-2a=0 \quad a(a-2)=0 \begin{cases} a=0 \; ✗ \\ a=2 \; ✓ \end{cases}$

سوال ۴:

$f=\{(3,5),(4,7),(7,2),(9,4)\}$ $\quad f^{-1}\{(5,3),(7,4),(2,7),(4,9)\}$

$g=\{(3,2),(8,4),(9,5)\}$ $\quad \{(2,3),(4,8),(5,9)\}$

$5\to3\to5 \quad 8\to9\to4$

الف) $f\circ f^{-1}$ $\quad 7\to4\to7$, $2\to7\to2$ $\quad \{(5,5),(7,7),(2,2),(4,4)\}$

ب) $f^{-1}\circ f$ $\quad 3\to5\to3$, $7\to2\to7$, $4\to7\to4$, $9\to4\to9$ $\quad \{(3,3),(4,4),(7,7),(9,9)\}$

۵

ج) $f\circ g^{-1}$ $\; f(g^{-1})$ $\quad 2\to3\to5$, $4\to8\to$, $5\to9\to4$ $\quad \{(2,5),(5,4)\}$

د) $g^{-1}\circ f$ $\quad g^{-1}(f)$ $\quad 3\to5\to9$, $9\to4\to8$, $4\to7\to$, $7\to2\to3$ $\quad \{(3,9),(7,3),(9,8)\}$

۹)

$$f(x) = x^2 - \frac{(x+1)^3}{x+3}$$

$$f^{-1}(x) = \frac{ax+b}{2x+d}$$

$$x^2(x+3) \to \frac{x^3+3x^2-x^3-3x^2-3x-1}{x+3}$$

$$\to \frac{-(3x+1)}{x+3} = y$$

$$f() = -\frac{(3x+1)}{x+3} \to f^{-1}(x) = -\frac{(3y+1)}{y+3}$$

$$\to xy + 3x = -3y - 1 \qquad (3+x)y = -3x - 1$$

$$a = -6 \quad d = 6 \quad b = -2 \leftarrow f(x)^{-1} = \frac{-6x-2}{2x+6} \xleftarrow{\times 2}$$

$$f^{-1}(-2) = \frac{-6\times(-2)-2}{2\times(-2)+6} = 5$$

$$f(x) = \frac{x}{x^2+1} \to y = \frac{x}{x^2+1} \to x = \frac{y}{y^2+1}$$ ۱۰)

$$xy^2 + x = y \qquad xy^2 - y + x = 0$$

$$xy^2 + x = y$$
$$xy^2 - y + x = 0$$

$$1 - 4x^2 \geq 0 \to 1 \geq 4x^2$$

$$\frac{1}{4} \geq x^2 \to -\frac{1}{2} \leq x \leq \frac{1}{2}$$

$$\to x \neq 0$$

$$\to f^{-1}(x) \begin{cases} \dfrac{1+\sqrt{1-4x^2}}{2x} & 0 < x \leq \dfrac{1}{2} \\ \dfrac{1-\sqrt{1-4x^2}}{2x} & -\dfrac{1}{2} \leq x < 0 \end{cases}$$