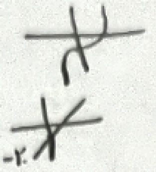
شهرزاد شوشتری ۱۱

$$f(x)=\begin{cases} x^3-4 & x>a \\ 12x-20 & x\le a\end{cases}$$ ۱)



$a^3-4>12a-20$

$a^3-12a+16>0 \to (a-2)(a^2+2a-8)>0 \quad (a-2)(a+4)(a-2)\ge 0$

$a\in(-4,+\infty)-\{2\}$

$f^{-1}(2)=4 \to f(4)=2$ ۲)

$f(x)=3x+k \to 12+k=2 \to k=-10 \to f(x)=3x-10$

$f(7)=21-10=11$

$f(f(x))=3(3x-10)-10=9x-30-10 \to 9x-40$

$f(x)=\dfrac{ax}{x-1} \xrightarrow{(a,2a)} \dfrac{a^2}{a-1}=2a \to a^2=2a^2-2a=0 \quad a^2-2a=0 \quad a(a-2)=0 \quad a=0 \quad \boxed{2}$ ۳)

غ ق ق

$g^{-1}=\{(2,3)(4,8)(5,9)\}$ ۴)

$f^{-1}=\{(5,3)(7,4)(2,7)(4,9)\}$

$f\circ f^{-1}=\{(5,5)(7,7)(2,2)(4,4)\}$

$f^{-1}\circ f=\{(3,3)(4,4)(7,7)(9,9)\}$

$f\circ g^{-1}=\{(2,5)(5,9)\}$

$g^{-1}\circ f=\{(3,9)(7,3)(4,8)\}$

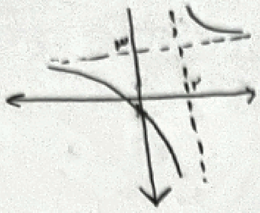
$f=\{(2,4)(4,8)(9,0)\} \quad g=\{(2,1)(4,3)(9,5)\} \quad h=\{(1,2)(3,4)(4,1)(5,1)\}$ ۵)

$g^{-1}=\{(1,2)(3,4)(5,9)\}$

$$\frac{h}{f\circ g^{-1}} = \cancel{\frac{2}{4}}_{\,\frac{1}{2}} ,\ \cancel{\frac{4}{8}}_{\,\frac{1}{2}} ,\ \left\{\left(1,\tfrac{1}{2}\right)\left(3,\tfrac{1}{2}\right)\right\}$$

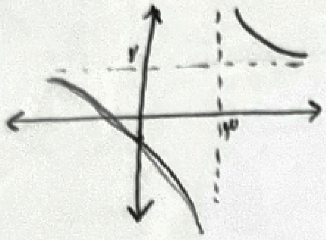
۶)

$y = \frac{3x+1}{x-2}$



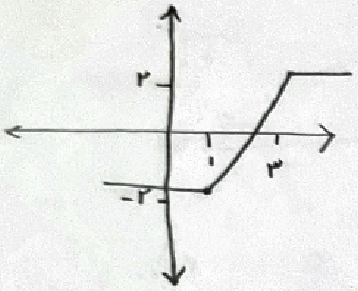
$\vee \rightarrow x = \frac{3y+1}{y-2} \rightarrow y = \frac{2x+1}{x-3}$

$x=0 \rightarrow \frac{1}{-2}$



$x=0 \rightarrow \frac{1}{-3}$

۷)



$\rightarrow [a,b] = [1,3] \quad f(x) = x-1+x-3 = 2x-4 \quad 1 \leq x \leq 3$

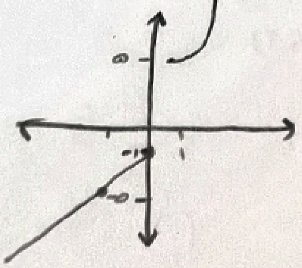
$y = 2x-4 \rightarrow x = 2y-4 \rightarrow y = \frac{x+4}{2} = \boxed{\frac{x}{2}+2}$

۸)

$f(x) = \begin{cases} x^3+4 & x \geq 1 \\ 4x-1 & x \leq 0 \end{cases}$

$4x-1 \rightarrow x=0 \rightarrow -1$

$x=-1 \rightarrow -5$



$y = x^3+4 \rightarrow x = y^3+4 \rightarrow \sqrt[3]{x-4} = y \quad x \geq 5$

$y = 4x-1 \rightarrow x = 4y-1, \; y = \frac{x+1}{4} \quad x \leq -1$

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$$f(x) = x^2 - \frac{(x+1)^3}{x+3} \rightarrow \frac{x^2(x+3) - (x+1)^3}{x+3} = \frac{\cancel{x^3} + \cancel{3x^2} - \cancel{x^3} - 1 - \cancel{3x^2} - 3x}{x+3} = \frac{-3x-1}{x+3}$$

$$y = \frac{-3x-1}{x+3} \rightarrow x = \frac{-3y-1}{y+3} \rightarrow y = \frac{-(3x+1)}{x+3} = \frac{-3x-1}{x+3} \frac{\times 2}{\times 2} \rightarrow \frac{-6x-2}{2x+6}$$

$$f^{-1}(-2) = \frac{-6\times(-2)-2}{2\times(-2)+6} = \frac{12-2}{4+6} = \frac{10}{2} = \boxed{5}$$

۱۰

$$f(x) = \frac{x}{x^2+1} \rightarrow x = \frac{y}{y^2+1} \rightarrow xy^2 + x = y \rightarrow xy^2 - y + x = 0$$

$$y = \frac{1 \pm \sqrt{1-4x^2}}{2x}$$