شهرزاد شوشتری ۱۱

۱۶,۷۵

۱) $f(x)=\begin{cases} x^3-4 & x>a \\ 12x-20 & x\le a \end{cases}$

۱,۷۵

$a^3-4 \geq 12a-20$

$a^3-12a+16\geq 0 \rightarrow (a-2)(a^2+2a-8)\geq 0 \quad (a-2)(a+4)(a-2)\geq 0 \quad \frac{-4 \quad 2}{-\ |\ +\ |\ +} \rightarrow$

$a\in[-4,+\infty)-\{2\}$

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۲) $f^{-1}(2)=4 \rightarrow f(4)=2$

$f(x)=3x+k \rightarrow 12+k=2 \rightarrow k=-10 \rightarrow f(x)=3x-10$

$f(7)=21-10=11$

$f(f(x))=3(3x-10)-10=9x-30-10 \rightarrow 9x-40$

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۳) $f(x)=\frac{ax}{x-1} \xrightarrow{(a,2a)} \frac{a^2}{a-1}=2a \rightarrow a^2=2a^2-2a=0 \quad a^2-2a=0 \quad a(a-2)=0 \quad a=0, 2$

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۴) $g^{-1}=\{(2,3)(4,8)(5,9)\}$

$f^{-1}=\{(5,3)(7,4)(3,7)(4,9)\}$

$fof^{-1}=\{(5,5)(7,7)(3,3)(4,4)\}$

$f^{-1}of=\{(3,3)(4,4)(7,7)(9,9)\}$

$fog^{-1}=\{(2,5)(5,9)\}$

$g^{-1}of=\{(3,9)(7,3)(4,8)\}$

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۵) $f=\{(2,4)(4,8)(9,0)\} \quad g=\{(2,1)(4,3)(9,5)\} \quad h=\{(1,2)(3,4)(4,1)(5,1)\}$

$g^{-1}=\{(1,2)(3,4)(5,9)\}$

$\frac{h}{fog^{-1}}=\frac{2}{4}=\frac{1}{2},\ \frac{4}{8}=\frac{1}{2},\ \{(1,\frac{1}{2})(3,\frac{1}{2})\}$

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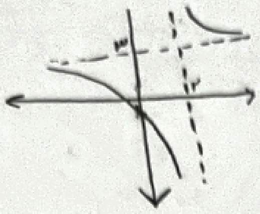
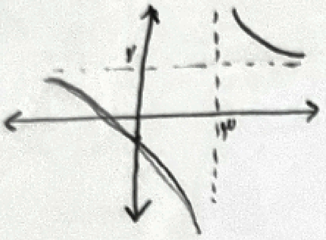
۶)

$y = \frac{3x+1}{x-2}$

$x=0 \to \frac{1}{-2}$



$\to x = \frac{3y+1}{y-2} \to y = \frac{2x+1}{x-3}$



$x=0 \to \frac{1}{-3}$

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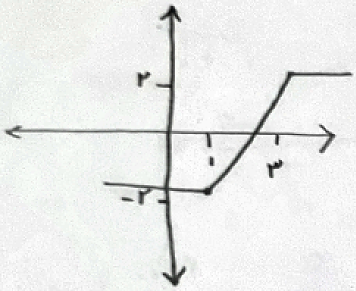
۷)



$\to [a,b] = [1,3]$ $\quad f(x) = x-1+x-3 = 2x-4 \quad 1 \le x \le 3$

$y = 2x-4 \to x = 2y-4 \to y = \frac{x+4}{2} = \boxed{\frac{x+4}{2}}$

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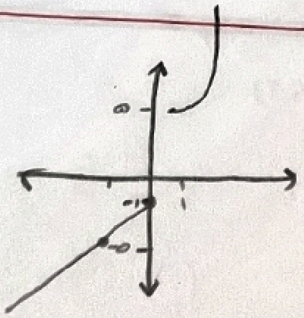
۸)

$f(x) = \begin{cases} x^3+4 & x \ge 1 \\ 4x-1 & x \le 0 \end{cases}$

$4x-1 \to x=0 \to -1$

$x=-1 \to -5$



$y = x^3+4 \to x = y^3+4 \to \sqrt[3]{x-4} = y \quad x \ge 5$

$y = 4x-1 \to x = 4y-1 , \; y = \frac{x+1}{4} \quad x \le -1$

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$$f(x) = x^{۲} - \frac{(x+۱)^{۳}}{x+۳} \rightarrow \frac{x^{۲}(x+۳) - (x+۱)^{۳}}{x+۳} = \frac{x^{۳} + ۳x^{۲} - x^{۳} - ۱ - ۳x^{۲} - ۳x}{x+۳} = \frac{-۳x-۱}{x+۳}$$

$$y = \frac{-۳x-۱}{x+۳} \rightarrow x = \frac{-۳y-۱}{y+۳} \rightarrow y = \frac{-(۳x+۱)}{x+۳} = \frac{-۳x-۱}{x+۳} \times \frac{۲}{۲} \rightarrow \frac{-۹x-۲}{۲x+۹}$$

$$f^{-۱}(-۲) = \frac{-۹\times(-۲)-۲}{۲\times(-۲)+۹} = \frac{۱۶}{۴+۹} = \frac{۱۶}{۱۳} \quad ۵$$

$$f(x) = \frac{x}{x^{۲}+۱} \rightarrow x = \frac{y}{y^{۲}+۱} \rightarrow xy^{۲}+x = y \rightarrow xy^{۲}-y+x = ۰$$

$$y = \frac{۱ \pm \sqrt{۱-۴x^{۲}}}{۲x}$$