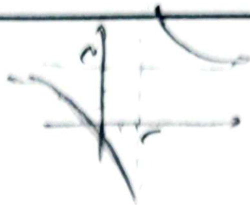
نام و نام خانوادگی ....[illegible].... پاسخنامه تشریحی تکلیف شماره .... کلاس ....

۱	$12a - 20 < a^3 - 4 \rightarrow 0 < a^3 - 12a + 16 \xrightarrow{+8-8} 0 < a^3 - 8 - 12a + 24$ $0 < (a-2)(a^2+2a+4) - 12(a-2)$ $0 < (a-2)(a^2+2a-8)$ $\underbrace{(a+4)(a-2)}_{\text{ریشه مضاعف}}$ $\boxed{-4 < a}$
۲	$f(x) = 2x+k \rightarrow f(6) = 12+k \rightarrow k=-10$ $\quad f(x) = 2x-10$ $D_{f^{-1}(x)} = R_{f(x)} \qquad R_{f^{-1}(x)} = D_{f(x)}$ الف) $f(7) = 11$ ب) $f(f(x)) = f(2x-10) = 2(2x-10)-10 = 9x-40$
۳	$f(x) = \frac{ax}{x-1} \rightarrow x = \frac{ay}{y-1} \rightarrow f^{-1}(x) = \frac{x}{x-a} \rightarrow a = \frac{2a}{a} \rightarrow \boxed{a=2}$ $\quad A(2a, a)$
۴	الف) $f \circ f^{-1} = f(f^{-1}(x)) = \varnothing$ ب) $f^{-1} \circ f = f^{-1}(f(x)) = \varnothing$ ج) $f \circ g^{-1} = \{(3,5),(5,6)\}$ د) $g^{-1} \circ f = \{(2,9)(7,2)(9,8)\}$
۵	$\frac{h}{f \circ g^{-1}}$ $f \circ g^{-1} = \{(1,4)(2,8),(5,6)\}$ $h = \{(1,3),(2,4),(4,1),(5,1)\}$ $\{(1, \frac{3}{\cancel{4}}), (2, \frac{4}{8}), (5, \frac{1}{6})\} = \{(1,\frac{1}{2}), (2,\frac{1}{2})\}$ تعریف شده $\frac{h}{f \circ g^{-1}} = \frac{1}{2}$

حل مسائل

۶

$y=\frac{2x+1}{x-2}$

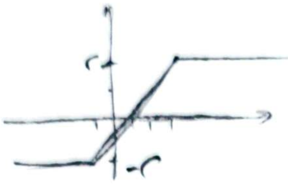


تابع یک به یک ✓

$x=\frac{2x+1}{x-2} \to x=\frac{2y+1}{y-2} \to f^{-1}(x)=\frac{2x+1}{x-2}$

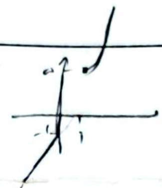
۷

$f(x)=|x-1|-|x-2| \qquad [a,b]=[-1,1]$



۸

$f(x)=\begin{cases} x^2+4 & x\ge 1 \\ 4x-1 & x\le 0 \end{cases}$



تابع یک به یک ✓

$D_{f^{-1}(x)}=R_{f(x)}$

$f^{-1}(x)=\begin{cases} y^2+4=x \to y=\sqrt{x-4} & 5\le x \\ 4y-1=x \to y=\frac{x+1}{4} & x\le -1 \end{cases}$

$1\le\sqrt{x-4} \to 5\le x$

$\frac{x+1}{4}\le 0 \to x\le -1$

۹

$f(x)=x^2-\frac{(x+1)^2}{x+2} \qquad f^{-1}(x)=\frac{ax+b}{cx+d} \qquad f^{-1}(b)=?$

$\frac{x^3+2x^2-(x^2+1^2+2x(x+1))}{x+2}=\frac{x^3+2x^2-x^2-1-2x^2-2x}{x+2}=\frac{-1-2x}{x+2}$

$f(x)=y=\frac{-1-2x}{x+2} \to x=\frac{-2y-1}{y+2} \to f^{-1}(x)=\frac{2x-1}{x-2} \qquad c=1 \to f^{-1}(b)=\frac{-4}{-4}=1$

۱۰

$f(x)=\frac{x}{x^2+1} \to y=\frac{x}{x^2+1} \to yx^2+y=x \to yx^2-x+y=0$

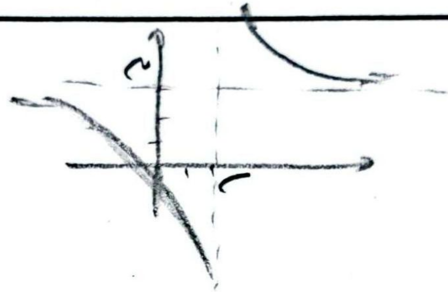
$\Delta=1-4y^2 \to x=\frac{1\pm\sqrt{1-4y^2}}{2} \qquad -\frac{1}{2}\le y\le\frac{1}{2}$

$f^{-1}(x)=y=\frac{1\pm\sqrt{1-4x^2}}{2} \qquad -\frac{1}{2}\le x\le\frac{1}{2}$

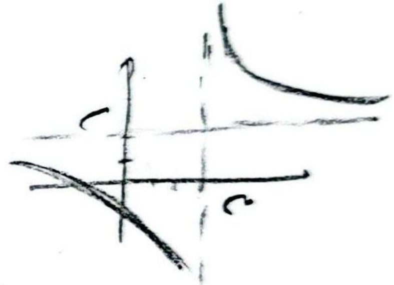
حمید جعفریان

یک به یک ✓

$y=\frac{۲x+۱}{x-۲}$

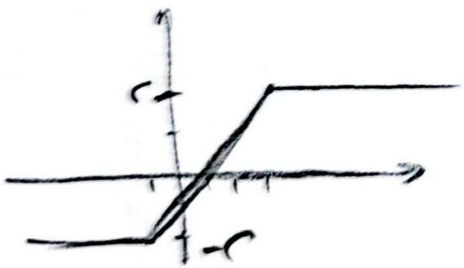


$y=\frac{۳x+۱}{x-۲} \Rightarrow x=\frac{۳y+۱}{y-۲} \rightarrow f^{-1}(x)=\frac{۲x+۱}{x-۳}$



۶

$f(x)=|x-1|-|x-۳|$ $[a,b]=[-۱,۲]$



۷