نام و نام خانوادگی : آیین عظیمی   تکلیف : ۱۱   کلاس : تجربی یازدهم

$$f(x)\begin{cases} x^3-4 & x>a \\ 12x-20 & x\le a \end{cases} \Rightarrow$$ همواره افزایشی هست → کمترین مقدار x^3-4 ≥ بیشترین مقدار $12x-20$

$a^3-4 \geq 12a-20$ ۱

$a^3-12a+16 \geq 0$

$a^3-4a-8a+16 \geq 0$

$a(a^2-4)-8(a-2) \geq 0$

$(a-2)(a^2+2a-8) \geq 0$

$(a-2)(a+4)(a-2) \geq 0$

$(a-2)^2(a+4) \geq 0$

$\begin{array}{c|ccc} & -4 & & 2 \\ \hline - & & + & + \end{array}$

$\boxed{a \in [-4,+\infty)}$

$f(x)=3x+k$, $f^{-1}(2)=4$ $\Big\} \to f^{-1}(2)=4 \to f(4)=2 \to 3(4)+k=2 \to f(x)=3x-10$

$12+k=2$

$\boxed{k=-10}$

الف) $f(7) \xrightarrow{3x-10} f(7)=21-10=11$ ۲

ب) $f(f(x)) \to f(3x-10) : 3(3x-10)-10 = 9x-30-10 = \boxed{9x-40}$

$A\begin{bmatrix} 2a \\ a \end{bmatrix} \to f^{-1}(2a)=a$

$f(x)=\dfrac{ax}{x-1}$ $f(a)=2a \Rightarrow f(a)=\dfrac{a^2}{a-1}=2a \to a^2=2a(a-1)$ ۳

$a^2=2a^2-2a$

$a^2-2a=0$

$a(a-2)=0$

$\begin{cases} a=0 & \text{غ ق ق} \to f(x)=0 \\ \boxed{a=2} & \end{cases}$

تابع ثابت است

$f:\{(3,5)(4,7)(7,2)(9,6)\} \to f^{-1}(x):\{(5,3)(7,4)(2,7)(6,9)\}$

$g:\{(3,2)(8,6)(9,5)\} \to g^{-1}(x):\{(2,3)(6,8)(5,9)\}$

الف) $f\circ f^{-1}:\{(5,5)(7,7)(2,2)(6,6)\}$ ج) $f\circ g^{-1}=\{(2,5)(\cancel{6})(5,6)\}$ ۴

ب) $f^{-1}\circ f=\{(3,3)(4,4)(7,7)(9,9)\}$ د) $g^{-1}\circ f=\{(3,9)(\cancel{4})(7,3)(9,8)\}$

$f=\{(2,4)(4,8)(6,10)\}$

$g=\{(2,1)(4,3)(6,5)\} \to g^{-1}(x)=\{(1,2)(3,4)(5,6)\}$

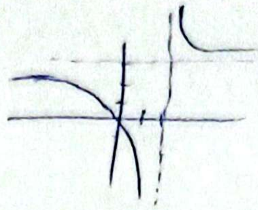
$\dfrac{h}{f\circ g^{-1}} \to f\circ g^{-1}:\{(1,4)(3,8)(5,10)\} \Rightarrow$ اشتراک دامنه : $\{1,3,5\}$ ۵

$h:\{(1,2)(3,4)(4,1)(5,1)\}$

$h=\{(1,2)(3,4)(4,1)(5,1)\}$

$\Rightarrow \dfrac{(1,2)(3,4)(5,1)}{(1,4)(3,8)(5,10)} = \boxed{\{(1,\tfrac{1}{2})(3,\tfrac{1}{2})\}}\ \cancel{(5,10)}$

$y = \dfrac{3x+1}{x-2}$ نمودار ✓ $\rightarrow$ مجانب افقی $= 3$ ، عمودی $= 2$ ، $f(0) = -\dfrac{1}{2}$

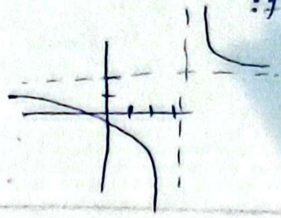


$: f(x)$

معکوس: $\dfrac{3y+1}{y-2} = x \rightarrow y = -\dfrac{-2x-1}{x-3} = \boxed{\dfrac{2x+1}{x-3} = y} : f^{-1}(x)$

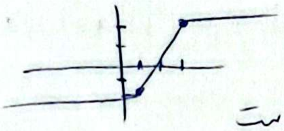
افقی $= 2$ ، عمودی $= 3$ ، $f(0) = -\dfrac{1}{3}$

$: f^{-1}(x)$ ۶



$f(x) = |x-1| - |x-3|$

$x \in [a,b]$



در بازه $[1,3] = [a,b]$ وارون پذیر است

$\rightarrow$ نقاط: $\begin{bmatrix}1\\-2\end{bmatrix}, \begin{bmatrix}3\\2\end{bmatrix}$ ۷

$m = \dfrac{2+2}{3-1} = \dfrac{4}{2} = 2$

$y - y_1 = m(x - x_1) \rightarrow y - 2 = 2(x-3)$

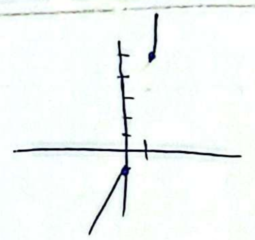
$f(x) = y = 2x - 4 \ , \ x \in [1,3]$

$R_f : [-2, 2]$

$f^{-1}(x) : x = 2y - 4$

$\boxed{\dfrac{x+4}{2} = y \ , \ x \in [-2,2]}$

$f(x) \begin{cases} x^3 + 4 & x \geq 1 \\ 4x - 1 & x \leq 0 \end{cases} \rightarrow$



نمودار ✓

$f(x): x^3 + 4$ $D: [1, +\infty)$ $R: [5, +\infty)$ $f^{-1}(x): x = y^3 + 4$, $y = \sqrt[3]{x-4}$

$f(x): 4x - 1$ $D: (-\infty, 0]$ $R: (-\infty, -1]$ $f^{-1}(x): x = 4y - 1$, $y = \dfrac{x+1}{4}$ ۸

$\Rightarrow f^{-1}(x) \begin{cases} \sqrt[3]{x-4} & x \geq 5 \\ \dfrac{x+1}{4} & x \leq -1 \end{cases}$

$\rightarrow \cancel{x^3} + \cancel{3x^2} - \cancel{x^3} - 1 - \cancel{3x^2} - 3x$

$f(x) = x^2 - \dfrac{(x+1)^3}{x+3} = \dfrac{x^3 + 3x^2 - (x^3 + 1 + 3x^2 + 3x)}{x+3} = \dfrac{-1-3x}{x+3}$

$\dfrac{b - dx}{2x - a} = \dfrac{-1-3x}{x+3} \ \ \times 2$ ۹

$f^{-1}(x) = \dfrac{ax+b}{2x+d} = y \rightarrow f(x): \dfrac{ay+b}{2y+d} = x \rightarrow y = -\dfrac{dx - b}{2x - a} = \dfrac{b - dx}{2x - a}$

$\dfrac{b - dx}{2x - a} = \dfrac{-2-6x}{2x+6}$

$b = -2$ ، $d = 6$ ، $a = -6$

$f^{-1}(-2) = \dfrac{-3(-2)-1}{-2+3} = 5 \leftarrow f^{-1}(x) = \dfrac{-3x-1}{x+3} \leftarrow f^{-1}(x) = \dfrac{-6x-2}{2x+6}$

$f(x) = \dfrac{x}{x^2+1} \rightarrow f^{-1}(x) : x = \dfrac{y}{y^2+1} \rightarrow xy^2 + x = y$

$xy^2 - y + x = 0$

$y = \dfrac{1 \pm \sqrt{1 - 4x^2}}{2x} \rightarrow$ شرط: $1 - 4x^2 \geq 0$ ۱۰

$4x^2 \leq 1$

$x^2 \leq \dfrac{1}{4}$

$-\dfrac{1}{2} \leq x \leq \dfrac{1}{2}$

$\boxed{f^{-1}(x) = \dfrac{1 + \sqrt{1 - 4x^2}}{2x} \ , \ x \in \left[-\dfrac{1}{2}, \dfrac{1}{2}\right] \ , \ x \neq 0}$