

١١ تبیین بقاء

$$f(x) = \begin{cases} x^k - \varepsilon & ; x > a \\ 1/x - \varepsilon & ; x \leq a \end{cases}$$

(1)

$$x^k - \varepsilon = 1/x - \varepsilon \rightarrow x^k - \varepsilon - \varepsilon = 1/x - \varepsilon - \varepsilon$$

$$\rightarrow x^k - 1 = 1/x - 2\varepsilon \rightarrow (x-1)(x^k + \varepsilon + 1/x) = 1/x(x-1)$$

$$(x-1)(x^k + \varepsilon + 1/x) = (x-1)(x^k + \varepsilon - 1/x)$$

$$\rightarrow (x-1)^2 (x^k + \varepsilon) \geq 0 \rightarrow \frac{-\varepsilon}{-1 + 1 + \varepsilon}$$

$$\Rightarrow a \geq -\varepsilon$$

الف)  $f(v) = ?$  (11)

(12)

$$f^{-1}(v) = \varepsilon \rightarrow f(\varepsilon) = \varepsilon \Rightarrow f(\varepsilon) = \underbrace{v \times \varepsilon}_{1/\varepsilon} + \varepsilon = v$$

$$\Rightarrow f(v) = v \times v - 1 = (11)$$

$$\rightarrow f(f(x)) \rightarrow f(x) = v_n - 1$$

$$\rightarrow f(f(x)) = v(v_n - 1) - 1 = \underline{v_n - \varepsilon}$$

$$f(x) = \frac{ax}{x-1}$$

(۳)

$$A(x, y) \rightarrow A^{-1}(y, x)$$

$$\rightarrow \frac{ax}{x-1} = y \Rightarrow ax = yx - y$$

$$\rightarrow ax - yx = -y \rightarrow x(a-y) = -y$$

$$a = y$$

چون باید دارد و نیز بار و در صورت و در  
منه می رود که ت. ن است!

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قبل قبل

الف)  $f \circ f$ 

$$f = \{(3, 8), (8, 7), (7, 2), (4, 4)\} \quad (۴)$$

ب)  $f^{-1} \circ f$ 

$$g = \{(3, 2), (1, 4), (4, 8)\}$$

ج)  $f \circ g^{-1}$ 

f

g

د)  $g^{-1} \circ f$ 

$$3 \rightarrow 8$$

$$3 \rightarrow 2$$

$$8 \rightarrow 7$$

$$1 \rightarrow 4$$

$$7 \rightarrow 2$$

$$4 \rightarrow 8$$

$$4 \rightarrow 4$$

f'

$$8 \rightarrow 3$$

$$7 \rightarrow 8$$

$$2 \rightarrow 7$$

$$4 \rightarrow 4$$

g^{-1}

$$2 \rightarrow 3$$

$$4 \rightarrow 1$$

$$8 \rightarrow 4$$

~~(a) f o f^{-1} = \{(1,1)(2,2)(3,3)(4,4)\}~~
 $\rightarrow f o f^{-1} = \{(1,1)(2,2)(3,3)(4,4)\}$

الف)  $f o f^{-1} = \{(0,0)(1,1)(2,2)(3,3)(4,4)\}$

ب)  $f o g^{-1} = \{(1,0)(2,4)\}$

ج)  $g o f^{-1} = \{(1,4)(2,3)(4,1)\}$

$f = \begin{matrix} 1 \rightarrow 1 \\ 2 \rightarrow 1 \\ 4 \rightarrow 0 \end{matrix}$ 
 $g^{-1} = \begin{matrix} 1 \rightarrow 1 \\ 3 \rightarrow 2 \\ 0 \rightarrow 4 \end{matrix}$

(a)

$h = \{(1,2)(3,2)(2,1)(0,1)\}$

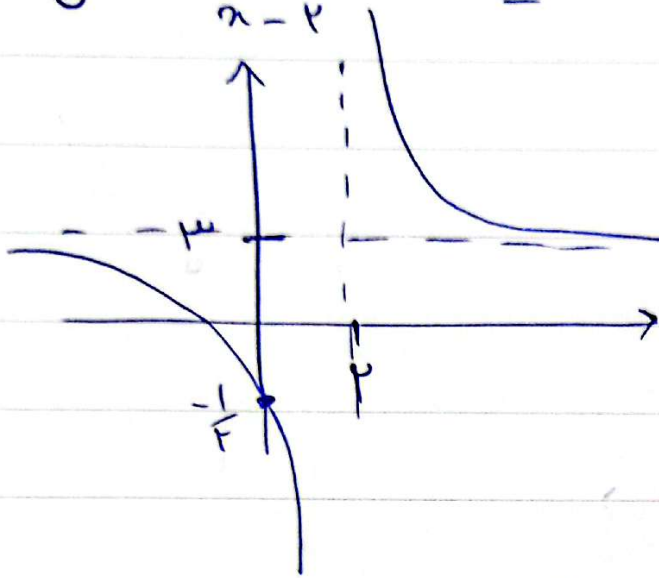
$\rightarrow f o g^{-1} = \{(1,2)(3,1)(0,0)\}$

$\frac{h}{f o g^{-1}} = \{(1, \frac{2}{1}), (3, \frac{1}{1}), (0, \frac{0}{0})\}$

$\Rightarrow \frac{h}{f o g^{-1}} = \{(1, \frac{1}{1}), (3, \frac{1}{1})\}$

$$y = \frac{pn+1}{n-p} \rightarrow \text{افضل} \Rightarrow \sqrt{\text{افضل}}$$

(4)



$$n = \frac{py+1}{y-p}$$

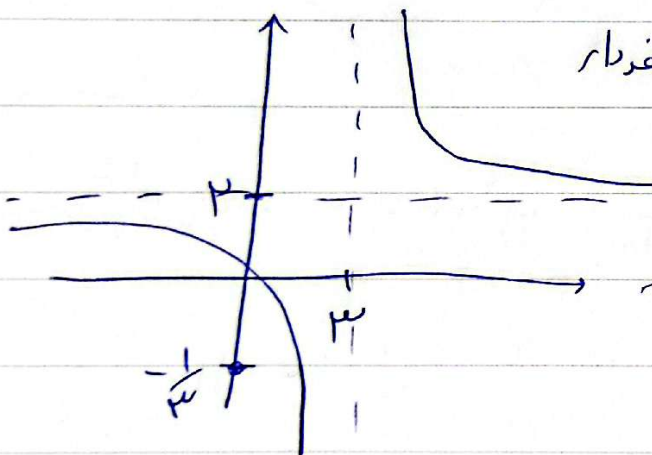
$$ny - pn = py + 1$$

$$ny - py = pn + 1$$

$$y(n-p) = pn+1$$

$$y = \frac{pn+1}{n-p}$$

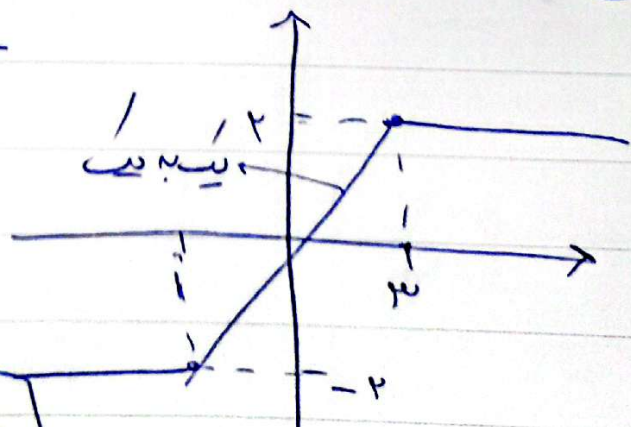
افضل



افضل

$$1 - x + x - p = \frac{x-1}{-p} = \frac{1-x}{p}$$

(5)



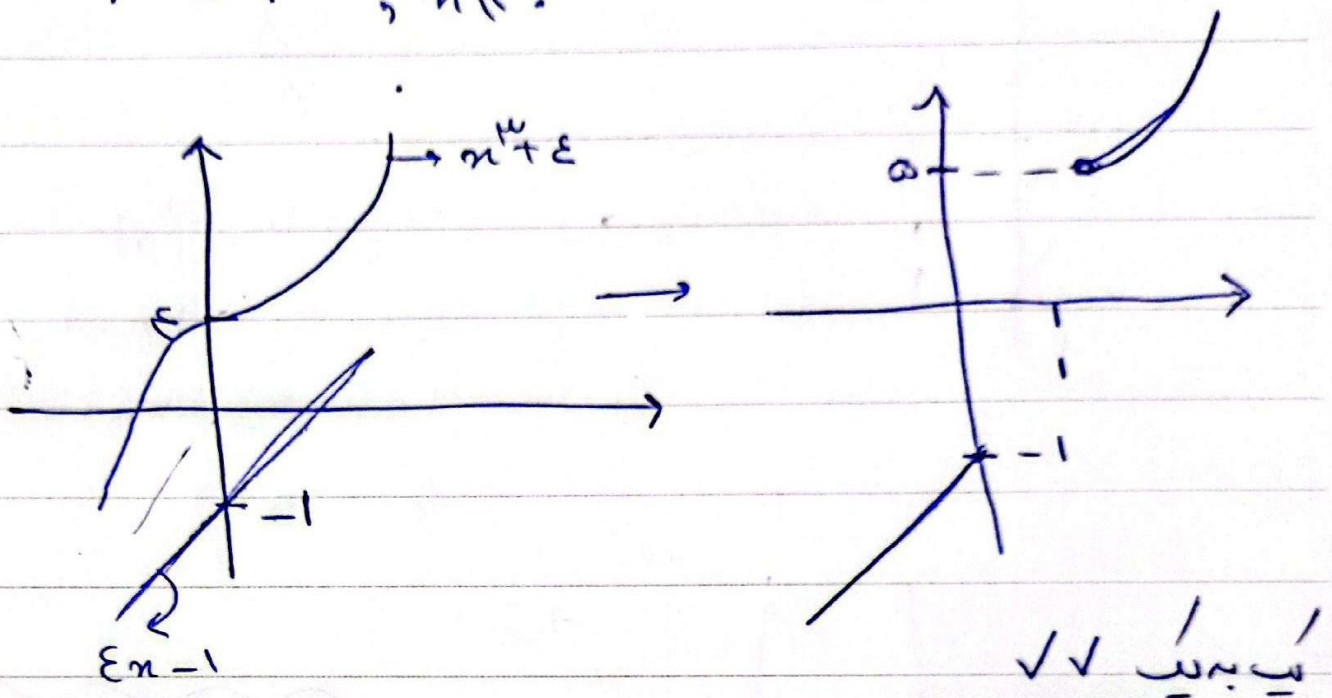
$$pn - \varepsilon = y \cdot \text{افضل}$$

$$n = py - \varepsilon \rightarrow$$

$$y = \frac{n+\varepsilon}{p}$$

$$f(x) = \begin{cases} x^3 + \varepsilon & ; x \geq 1 \\ \varepsilon x - 1 & ; x \leq 0 \end{cases}$$

①



$$x^3 + \varepsilon = y \xrightarrow{\text{حل}} y^3 + \varepsilon \leq x - y = \sqrt[3]{x - \varepsilon}$$

$$\rightarrow y = \varepsilon x - 1 \xrightarrow{\text{حل}} x = \varepsilon y - 1 \Rightarrow y = \frac{x+1}{\varepsilon}$$

$$\rightarrow 0 \leq x \rightarrow x \geq 1 \Rightarrow x^3 + \varepsilon \rightarrow 1 + \varepsilon \geq 0$$

$$\Rightarrow \underline{x \geq 1} \text{ ①}, \quad \varepsilon x - 1; x \leq 0 \Rightarrow 0 - 1 = -1$$

$$\Rightarrow \underline{x \leq -1} \text{ ②}$$

$$\Rightarrow f^{-1}(x) = \begin{cases} y = \sqrt[3]{x - \varepsilon} & ; x \geq 0 \\ y = \frac{x+1}{\varepsilon} & ; x \leq -1 \end{cases}$$

$$f(x) = x^2 - \frac{(x+1)^2}{x+2}$$

(9)

$$\rightarrow y = \frac{x^2 + \cancel{2x} - \cancel{x^2} - \cancel{2x} - \cancel{1}}{x+2} = \frac{-x-1}{x+2}$$

$$\rightarrow y = \frac{-x-1}{x+2} \Rightarrow x = \frac{-2y-1}{y+2} \rightarrow xy + 2x = -2y-1$$

$$\rightarrow xy + 2x = -2y-1 \rightarrow y(x+2) = -2y-1$$

$$\rightarrow y = \frac{-2y-1}{x+2} \Rightarrow b = -2 \quad f^{-1}(b) \rightarrow \frac{-2(-2)-1}{-2+2} = \frac{4-1}{0} = \frac{3}{0} = \infty$$

$$f(x) = \frac{x}{x^2+1}$$

(10)

$$\rightarrow yx^2 + y = x$$

$$yx^2 - x + y = 0 \rightarrow \frac{x \pm \sqrt{1-4y^2}}{2y} = y \Rightarrow y' = \frac{1 \pm \sqrt{1-4y^2}}{2y}$$

$$\Rightarrow y' = \frac{1 + \sqrt{1-4y^2}}{2y} \quad \text{و} \quad x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$$

$$1-4y^2 \geq 0 \rightarrow 1 \geq 4y^2 \Rightarrow \frac{1}{2} \geq |y| \geq -\frac{1}{2}$$