

$$x^2 - 4 \rightarrow \text{معادله}$$

$$12x - 20 \rightarrow \text{معادله (تعیین کننده)}$$

$$x \leq a \rightarrow 12a - 20 < a^2 - 4 \rightarrow$$

$$\frac{-4}{+} \quad \frac{2}{-} \quad \frac{2}{+}$$

$$\Rightarrow a \in \mathbb{R} - [-4, 2]$$

$$f(x) = 3x + k$$

$$f^{-1}(2) = 4$$

$$f(v) = 3(v) - 12 = 8 \text{ است}$$

$$x \rightarrow f(x) = 2$$

$$3(x) + k = 2 \Rightarrow k = -10 \Rightarrow f(x) = 3x - 12$$

$$b) \quad 3(3x - 12) - 12 = 9x - 48$$

$$f(a) = \frac{a^2}{a-1} = 2a$$

$$\rightarrow a^2 = 2a^2 + 2a \Rightarrow -a^2 + 2a = 0 \Rightarrow a(a-2) = 0 \Rightarrow a \in \{0, 2\}$$

اگر $(2a, a)$ می باشد پس $(a, 2a)$ در تابع مستقیم است.

$$f = \{(3, 5), (2, 7), (4, 9), (1, 4)\}$$

$$f^{-1} = \{(5, 3), (7, 2), (9, 4), (4, 1)\}$$

$$g = \{(3, 2), (1, 4), (9, 5)\}$$

$$g^{-1} = \{(2, 3), (4, 1), (5, 9)\}$$

$$a) \quad f \circ f^{-1} = \{(5, 5), (7, 7), (9, 9), (4, 4)\} \quad b) \quad f^{-1} \circ f = \{(3, 3), (2, 2), (4, 4), (1, 1)\}$$

$$c) \quad f \circ g^{-1} = \{(2, 5), (5, 4)\}$$

$$d) \quad g^{-1} \circ f = \{(3, 9), (7, 3), (9, 8)\}$$

$$g^{-1} = \{(1, 2), (3, 4), (5, 9)\}$$

$$f \circ g^{-1} = \{(1, 4), (3, 8), (5, 81)\} \quad h = \{(1, 2), (3, 4), (5, 9), (4, 1)\}$$

$$\frac{h}{f \circ g^{-1}} = \left\{ \left(1, \frac{4}{5}\right), \left(3, \frac{8}{5}\right), \left(5, \frac{81}{5}\right) \right\}$$

$$y = \frac{2x+1}{x-2}$$

$$\text{مشتق: } x > 2$$

$$x < 2 \rightarrow y < -\frac{1}{4}$$

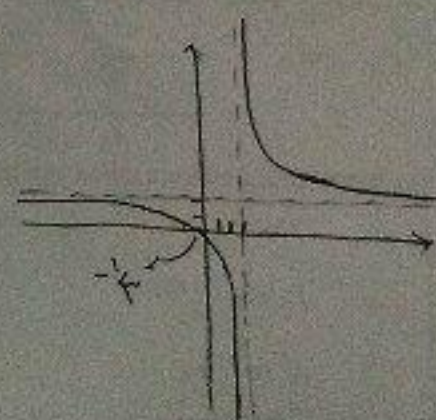
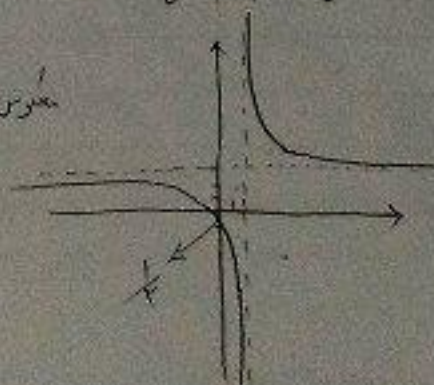
$$x > \frac{2y+1}{y-2} \rightarrow y = \frac{2x+1}{x-2}$$

$$\text{مشتق: } x > 2$$

$$\text{مشتق: } y > 2$$

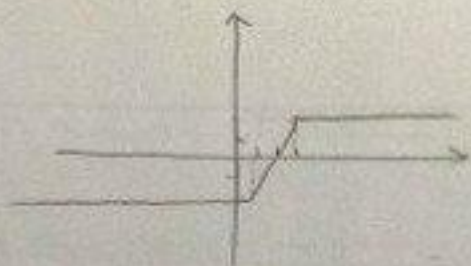
$$x < 2 \rightarrow y < -\frac{1}{4}$$

مشتق پذیری



✓

$$\begin{cases} f & x > 1 \\ x-f & 1 < x \leq 1 \\ -f & x < 1 \end{cases}$$

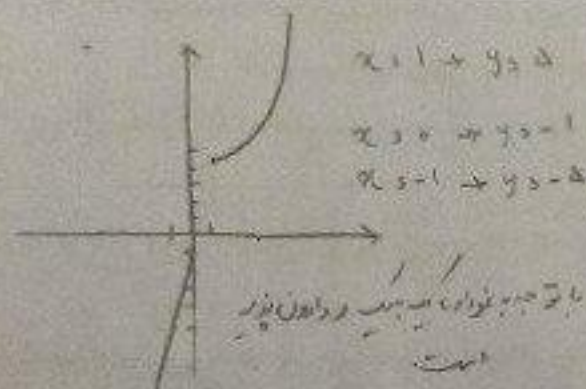


$$\Rightarrow [a, b] = [1, 1]$$

$$\text{مثلاً: } y = \ln x \rightarrow x = e^y = e$$

$$\Rightarrow \frac{x+f}{x} = y$$

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$$x=1 \rightarrow y=0$$

$$x=0 \rightarrow y=-1$$

$$x=-1 \rightarrow y=-2$$

$$\text{مثلاً: } y = x^2 + 1 \rightarrow x = y^2 + 1$$

$$\Rightarrow f(x) = \sqrt{x-1}, x \geq 1$$

$$\text{مثلاً: } -1 < x < y \rightarrow y = x + 1$$

$$\Rightarrow y = \frac{x+1}{2}, x \leq 0$$

$$\text{مثلاً: } \begin{cases} \sqrt{x-1} & x \geq 1 \\ \frac{x+1}{2} & x \leq 0 \end{cases}$$

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$$y = x^2 - \frac{(x+1)^2}{x+1} \rightarrow x = y^2 - \frac{(y+1)^2}{y+1} = \frac{y^2 - y^2 - 2y - 1}{y+1} = \frac{-2y-1}{y+1}$$

$$\rightarrow y = \frac{-2x-1}{x-1} = \frac{2x+1}{x-1} \rightarrow a=-2, b=1, d=-1 \rightarrow f^{-1}(-1) = \frac{-2(-1)-1}{-1-1} = \frac{1}{-2} = -\frac{1}{2}$$

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$$y = \frac{x}{x^2+1} \rightarrow x = \frac{y}{y^2+1} \rightarrow xy^2 - y + x = 0 \rightarrow x = \frac{1 \pm \sqrt{1-4y^2}}{2y}$$

$$1-4y^2 \geq 0 \rightarrow |y| \leq \frac{1}{2} \rightarrow -\frac{1}{2} \leq y \leq \frac{1}{2} \rightarrow \text{محدود است}$$

$$f(x) = \begin{cases} \frac{1 - \sqrt{1-4x^2}}{2x} & x > 0 \\ \frac{1 + \sqrt{1-4x^2}}{2x} & x < 0 \end{cases}$$